1. The equation \( x^2 - 3x + k^2 = 4 \) has two distinct real roots. Find the possible values of \( k \).

**Markscheme**

- evidence of rearranged quadratic equation (may be seen in working) \( A1 \)
  
  - \( e.g. \quad x^2 - 3x + k^2 - 4 = 0 \), \( k^2 - 4 \)

- evidence of discriminant (must be seen explicitly, not in quadratic formula) \( (M1) \)
  
  - \( e.g. \quad b^2 - 4ac \), \( \Delta = (-3)^2 - 4(1)(k^2 - 4) \)

- recognizing that discriminant is greater than zero (seen anywhere, including answer) \( R1 \)
  
  - \( e.g. \quad b^2 - 4ac > 0 \), \( 9 + 16 - 4k^2 > 0 \)

- correct working (accept equality) \( A1 \)
  
  - \( e.g. \quad 25 - 4k^2 > 0 \), \( 4k^2 < 25 \), \( k^2 = \frac{25}{4} \)

- both correct values (even if inequality never seen) \( (A1) \)
  
  - \( e.g. \quad \pm \sqrt{\frac{25}{4}} \), \( \pm 2.5 \)

- correct interval \( A1 \) \( N3 \)
  
  - \( e.g. \quad -\frac{5}{2} < k < \frac{5}{2} \), \( -2.5 < k < 2.5 \)

**Note:** Do not award the final mark for unfinished values, or for incorrect or reversed inequalities, including \( \leq \), \( k > -2.5 \), \( k < 2.5 \).

**Special cases:**

- If working shown, and candidates attempt to rearrange the quadratic equation to equal zero, but find an incorrect value of \( c \), award \( A1M1R1A0A0A0 \).

- If working shown, and candidates do not rearrange the quadratic equation to equal zero, but find \( c = k^2 \) or \( c = \pm 4 \), award \( A0M1R1A0A0A0 \).

**[6 marks]**

**Examiners report**

The majority of candidates who attempted to answer this question recognized the need to use the discriminant, however very few were able to answer the question successfully. The majority of candidates did not recognize that the quadratic equation must first be set equal to zero. In addition, many candidates simply set their discriminant equal to zero, instead of setting it greater than zero. Even many of the strongest candidates, who obtained the correct numerical values for \( k \), were unable to write their final answers as a correct interval.

This question is a good example of candidates who reach for familiar methods, without really thinking about what the question is asking them to find. There were many candidates who attempted to solve for \( x \) using the quadratic formula or factoring, even though the question did not ask them to solve for \( x \).
Let \( f(x) = x^3 - 2x - 4 \). The following diagram shows part of the curve of \( f \).

2a. Write down the \( x \)-coordinate of \( P \). [1 mark]

**Markscheme**

\( x = 2 \) (accept \((2, 0)\))  \( AI \ N1 \)

[1 mark]

**Examiners report**

This question was generally done well.

2b. Write down the gradient of the curve at \( P \). [2 marks]

**Markscheme**

evidence of finding gradient of \( f \) at \( x = 2 \)  \( (M1) \)

e.g. \( f'(2) \)

the gradient is 10  \( AI \ N2 \)

[2 marks]

**Examiners report**

Most candidates did not use their GDC in part (b), resulting in a variety of careless errors occasionally arising either in differentiating or substituting.

2c. Find the equation of the normal to the curve at \( P \), giving your equation in the form \( y = ax + b \). [3 marks]
**Markscheme**

evidence of negative reciprocal of gradient  \((MI)\)

e.g. \(\frac{-1}{f'(x)} = \frac{-1}{10}\)

evidence of correct substitution into equation of a line  \((AI)\)

e.g. \(y - 0 = \frac{1}{10} (x - 2)\), \(0 = -0.1(2) + b\)
\(y = -\frac{1}{10}x + \frac{2}{10}\)  \((accept a = -0.1\, , \, b = 0.2)\)  \(A1\)  \(N2\)

[3 marks]

**Examiners report**

There were some candidates who did not know the relationship between gradients of perpendicular lines while others found the equation of the tangent rather than the normal in part (c).

Let \(f(x) = a \cos(b(x - c))\) . The diagram below shows part of the graph of \(f\), for \(0 \leq x \leq 10\).

![Graph](image)

The graph has a local maximum at \(P(3, 5)\), a local minimum at \(Q(7, -5)\), and crosses the \(x\)-axis at \(R\).

3a. Write down the value of  \[2 marks\]

(i) \(a\) ;

(ii) \(c\).

**Markscheme**

(i) \(a = 5\)  \((accept -5)\)  \(A1\)  \(N1\)

(ii) \(c = 3\)  \((accept c = 7, \, if \, a = -5)\)  \(A1\)  \(N1\)

**Note:** Accept other correct values of \(c\), such as 11, -5, etc.

[2 marks]

**Examiners report**

Part (a) (i) was well answered in general. There were more difficulties in finding the correct value of the parameter \(c\).

3b. Find the value of \(b\).  \[2 marks\]
Markscheme

attempt to find period $\text{(M1)}$

e.g. $8, \ b = \frac{2\pi}{\text{period}}$

0.785398... 

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}, 0.785 [0.785, 0.786]$ (do not accept 45) $\text{A1 N2}$

[2 marks]

Examiners report

Finding the correct value of $b$ in part (b) also proved difficult as many did not realize the period was equal to 8.

3c. Find the $x$-coordinate of $R$. $\text{[2 marks]}$

Markscheme

valid approach $\text{(M1)}$

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept $(5, 0)$) $\text{A1 N2}$

[2 marks]

Examiners report

Most candidates could handle part (c) without difficulties using their GDC or working with the symmetry of the curve although follow through from errors in part (b) was often not awarded because candidates failed to show any working by writing down the equations they entered into their GDC.

The diagram below shows part of the graph of $f(x) = (x - 1)(x + 3)$.

4. (a) Write down the $x$-intercepts of the graph of $f$. $\text{[6 marks]}$

(b) Find the coordinates of the vertex of the graph of $f$. 
**Markscheme**

(a) \( x = 1, x = -3 \) (accept \((1, 0), (-3, 0)\)) \( A1A1 \quad N2 \)

[2 marks]

(b) **METHOD 1**

attempt to find \( x \)-coordinate \( (M1) \)

\[ eg \quad \frac{1\pm\sqrt{5}}{2}, x = \frac{1}{2}, f'(x) = 0 \]

correct value, \( x = -1 \) (may be seen as a coordinate in the answer) \( A1 \)

attempt to find their \( y \)-coordinate \( (M1) \)

\[ eg \quad f(-1), -2 \times 2, y = \frac{-D}{4a} \]

\( y = -4 \) \( A1 \)

vertex \((-1, -4)\) \( N3 \)

**METHOD 2**

attempt to complete the square \( (M1) \)

\[ eg \quad x^2 + 2x + 1 - 1 - 3 \]

attempt to put into vertex form \( (M1) \)

\[ eg \quad (x + 1)^2 - 4, (x - 1)^2 + 4 \]

vertex \((-1, -4)\) \( A1A1 \quad N3 \)

[4 marks]

**Examiners report**

Most candidates recognized the values of the \( x \)-intercepts from the factorized form of the function. Candidates also showed little difficulty finding the vertex of the graph, and employed a variety of techniques: averaging \( x \)-intercepts, using \( x = \frac{b}{2a} \), completing the square.

Let \( f(x) = \sqrt{x - 5} \), for \( x \geq 5 \).

5a. Find \( f^{-1}(2) \). 

[3 marks]

**Markscheme**

**METHOD 1**

attempt to set up equation \( (M1) \)

\[ eg \quad 2 = \sqrt{y - 5}, 2 = \sqrt{x - 5} \]

correct working \( (A1) \)

\[ eg \quad 4 = y - 5, x = 2^2 + 5 \]

\( f^{-1}(2) = 9 \) \( A1 \quad N2 \)

**METHOD 2**

interchanging \( x \) and \( y \) (seen anywhere) \( (M1) \)

\[ eg \quad x = \sqrt{y - 5} \]

correct working \( (A1) \)

\[ eg \quad x^2 = y - 5, y = x^2 + 5 \]

\( f^{-1}(2) = 9 \) \( A1 \quad N2 \)

[3 marks]
Examiners report
Candidates often found an inverse function in which to substitute the value of $x$. Some astute candidates set the function equal to $f(x)$ and solved for $x$. Occasionally a candidate misunderstood the notation as asking for a derivative, or used $\frac{1}{f(x)}$.

Examiners report
For part (b), many candidates recognized that if $g(30) = 3$ then $g^{-1}(3) = 30$, and typically completed the question successfully. Occasionally, however, a candidate incorrectly answered $\sqrt{25} = \pm 5$.

 marks
Examiners report
Part (b) proved challenging for most candidates, with few recognizing that changing 8 to base 2 is a helpful move. Some made it as far as $2^{\log_2{8}}$ yet could not make that final leap to an integer.

6. Find the value of $8^{\log_2{5}}$.

Markscheme
attempt to write 8 as a power of 2 (seen anywhere)  (M1)
$8 = 2^3$
$2^{3\log_2{5}} = 5^3$
$2^{3a} = 125$
$A1 \ N3$
[4 marks]

Examiners report
Part (b) proved challenging for most candidates, with few recognizing that changing 8 to base 2 is a helpful move. Some made it as far as $2^{\log_2{8}}$ yet could not make that final leap to an integer.

Consider $f(x) = \ln(x^4 + 1)$.

7a. Find the value of $f(0)$.

[2 marks]
Markscheme

substitute 0 into \( f \) \((M1)\)

e.g. \( \ln(0 + 1) , \ln 1 \)

\( f(0) = 0 \quad A1 \quad N2 \)

\([2 \text{ marks}]\)

Examiners report

Many candidates left their answer to part (a) as \( \ln 1 \). While this shows an understanding for substituting a value into a function, it leaves an unfinished answer that should be expressed as an integer.

The second derivative is given by \( f''(x) = \frac{4x^9(3-x^9)}{(x^8+1)^2} \).

The equation \( f''(x) = 0 \) has only three solutions, when \( x = 0 , \pm \sqrt[3]{3} (\pm 1.316 \ldots) \).

7b. There is a point of inflexion on the graph of \( f \) at \( x = \sqrt[3]{3} (x = 1.316 \ldots) \). Sketch the graph of \( f \), for \( x \geq 0 \). \([3 \text{ marks}]\)

Markscheme

\[ f(x) = 3 \sqrt[3]{4} (x = 1.316 \ldots) \]

Notes: Award \( A1 \) for shape concave up left of POI and concave down right of POI.

Only if this \( A1 \) is awarded, then award the following:

\( A1 \) for curve through \((0, 0)\), \( A1 \) for increasing throughout.

Sketch need not be drawn to scale. Only essential features need to be clear.

\([3 \text{ marks}]\)

Examiners report

Few candidates created a correct graph from the information given or found in the question. This included the point \((0, 0)\), the fact that the function is always increasing for \( x > 0 \), the concavity at \( x = 1 \) and the change in concavity at the given point of inflexion. Many incorrect attempts showed a graph concave down to the right of \( x = 0 \), changing to concave up.
The velocity of a particle in m s\(^{-1}\) is given by \(v = e^{\sin t} - 1\), for \(0 \leq t \leq 5\).

8a. On the grid below, sketch the graph of \(v\).

![Graph of v](image)

**Markscheme**

Note: Award A1 for approximately correct shape crossing x-axis with \(3 < x < 3.5\).

Only if this A1 is awarded, award the following:

A1 for maximum in circle, A1 for endpoints in circle.

[3 marks]

**Examiners report**

There was a minor error on this question, where the units for velocity were given as m s\(^{-2}\) rather than m s\(^{-1}\). Examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were considered at the grade award meeting.

Candidates continue to produce sloppy graphs resulting in loss of marks. Although the shape was often correctly drawn, students were careless when considering the domain and other key features such as the root and the location of the maximum point.

8b. Find the total distance travelled by the particle in the first five seconds.

**Markscheme**

\(t = \pi\) (exact), 3.14 A1 N1

[1 mark]

**Examiners report**

The fact that most candidates with poorly drawn graphs correctly found the root in (b)(i), clearly emphasized the disconnect between geometric and algebraic approaches to problems.
Let $f$ and $g$ be functions such that $g(x) = 2f(x + 1) + 5$.

9. (a) The graph of $f$ is mapped to the graph of $g$ under the following transformations: 

vertical stretch by a factor of $k$, followed by a translation $\left( \frac{p}{q} \right)$.

Write down the value of 

(i) $k$; 
(ii) $p$; 
(iii) $q$.

(b) Let $h(x) = -g(3x)$. The point $A(6, 5)$ on the graph of $g$ is mapped to the point $A'$ on the graph of $h$. Find $A'$.

**Markscheme**

(a) (i) $k = 2$ \(\text{AI N1}\) 
(ii) $p = -1$ \(\text{AI N1}\) 
(iii) $q = 5$ \(\text{AI N1}\)  
[3 marks]

(b) recognizing one transformation \(\text{(M1)}\) 
eg horizontal stretch by \(\frac{1}{3}\), reflection in x-axis 
$A'$ is $(2, -5)$ \(\text{AIA1 N3}\)  
[3 marks]

**Total [6 marks]**

**Examiners report**

Part (a) was frequently done well but a lack of understanding of the notation $f(x + 1)$ often led to an incorrect value for $p$. In part (b), candidates did not recognize the simplicity of the problem. Most seemed to be unable to correctly recognize the two transformations implied in the question and were thus unable to attempt a geometric solution. Flawed algebraic approaches to part (b) were common and many could not interpret the notation $g(3x)$ as multiplying the $x$-value by $\frac{1}{3}$.

Let $f(x) = \frac{100}{(1 + 50e^{-0.2x})}$. Part of the graph of $f$ is shown below.

10a. Write down $f(0)$.  
[1 mark]
Markscheme
\[ f(0) = \frac{100}{51} \text{ (exact), 1.96} \quad AI \quad NI \]
[1 mark]

Examiners report
Candidates had little difficulty with parts (a), (b) and (c).

10b. Solve \( f(x) = 95 \). 2 marks

Markscheme
setting up equation \((MI)\)
\[ eg \quad 95 = \frac{100}{1+50e^{-0.2x}} \quad \text{sketch of graph with horizontal line at } y = 95 \]
\[ x = 34.3 \quad AI \quad N2 \]
[2 marks]

Examiners report
Candidates had little difficulty with parts (a), (b) and (c). Successful analytical approaches were often used in part (b) but again, this was not the most efficient or expected method.

10c. Find the range of \( f \). 3 marks

Markscheme
upper bound of \( y \) is 100 \((AI)\)
lower bound of \( y \) is 0 \((AI)\)
range is 0 < \( y \) < 100 \( AI \quad N3 \)
[3 marks]

Examiners report
Candidates had little difficulty with parts (a), (b) and (c). In part (c), candidates gained marks by correctly identifying upper and lower bounds but often did not express them properly using an appropriate notation.

Let \( f(x) = 4x - 2 \) and \( g(x) = -2x^2 + 8 \).

11a. Find \( f^{-1}(x) \). 3 marks
Markscheme
interchanging $x$ and $y$ (seen anywhere) $(MI)$

$eg\ x = 4y - 2$

evidence of correct manipulation $(AI)$

$eg\ x + 2 = 4y$

$f^{-1}(x) = \frac{x+2}{4}$ (accept $y = \frac{x+2}{4}$, $\frac{x+2}{4}$, $f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$ $AI$ $N2$

[3 marks]

Examiners report
The overwhelming majority of candidates answered both parts of this question correctly. There were a few who seemed unfamiliar with the inverse notation and answered part (a) with the derivative or the reciprocal of the function.

11b. Find $(f \circ g)(1)$.

Markscheme
METHOD 1
attempt to substitute 1 into $g(x)$ $(MI)$

$eg\ g(1) = -2 \times 1^2 + 8$

$g(1) = 6$ $(AI)$

$f(6) = 22$ $AI$ $N3$

METHOD 2
attempt to form composite function (in any order) $(MI)$

$eg\ (f \circ g)(x) = 4(-2x^2 + 8) - 2 \ (= -8x^2 + 30)$

correct substitution

$eg\ (f \circ g)(1) = 4(-2 \times 1^2 + 8) - 2, -8 + 30$

$f(6) = 22$ $AI$ $N3$

[3 marks]

Examiners report
The overwhelming majority of candidates answered both parts of this question correctly. A few candidates made arithmetic errors in part (b) which kept them from finding the correct answer.
The diagram below shows the graph of a function $f$, for $-1 \leq x \leq 2$.

12a. Write down the value of $f(2)$.  

**Markscheme**

$f(2) = 3$  

A1 N1  

[1 mark]

**Examiners report**

In part (a) of this question, most candidates were able to find the value of $f(2)$ correctly, while some had trouble finding $f^{-1}(-1)$. Many candidates tried to find an equation for the function, or to make tables of values to help them find their answers. The intent of this question was to read the answers from the given graph. Candidates should be reminded that when the command term is "write down", there is no need for them to do large amounts of working.

12b. Write down the value of $f^{-1}(-1)$.

**Markscheme**

$f^{-1}(-1) = 0$  

A2 N2  

[2 marks]

**Examiners report**

In part (a) of this question, most candidates were able to find the value of $f(2)$ correctly, while some had trouble finding $f^{-1}(-1)$. Many candidates tried to find an equation for the function, or to make tables of values to help them find their answers. The intent of this question was to read the answers from the given graph. Candidates should be reminded that when the command term is "write down", there is no need for them to do large amounts of working.
12c. Sketch the graph of $f^{-1}$ on the grid below. [3 marks]

**Markscheme**

EITHER

- attempt to draw $y = x$ on grid  \((M1)\)

OR

- attempt to reverse $x$ and $y$ coordinates  \((M1)\)

  eg  writing or plotting at least two of the points

  \((-2, -1), (-1, 0), (0, 1), (3, 2)\)

THEN

- correct graph  \(A2\)  \(N3\)

**Examiners report**

In part (b) of this question, candidates were generally successful in reversing the $x$ and $y$ coordinates of key points or reflecting in the $y = x$ line to correctly sketch the graph of the inverse function. Common errors included not sketching the graph for the appropriate domain, or sketching the graph of $f(-x)$ or the graph of $-f(x)$. [3 marks]
Let \( f(x) = \sin x + \frac{1}{2} x^2 - 2x \), for \( 0 \leq x \leq \pi \).

Let \( g \) be a quadratic function such that \( g(0) = 5 \). The line \( x = 2 \) is the axis of symmetry of the graph of \( g \).

13a. Find \( g(4) \).

**Markscheme**

recognizing \( g(0) = 5 \) gives the point \((0, 5)\) \((R1)\)

recognize symmetry \((M1)\)

eg vertex, sketch

\[
g(4) = 5 \quad A1 \quad N3
\]

**Examiners report**

In part (b), many candidates did not understand the significance of the axis of symmetry and the known point \((0, 5)\), and so were unable to find \( g(4) \) using symmetry. A few used more complicated manipulations of the function, but many algebraic errors were seen.

The function \( g \) can be expressed in the form \( g(x) = a(x - h)^2 + 3 \).

13b. (i) Write down the value of \( h \).

(ii) Find the value of \( a \).
**Markscheme**

(i)  \( h = 2 \quad A1 \quad N1 \)

(ii) substituting into \( g(x) = a(x^2 + 3) \) (not the vertex) \( (MI) \)

\[
\begin{align*}
5 &= a(0 - 2)^2 + 3, \\
5 &= a(4 - 2)^2 + 3
\end{align*}
\]

\( eg \quad 5 = 4a + 3, 4a = 2 \)

\( a = \frac{1}{2} \quad A1 \quad N2 \)

[4 marks]

**Examiners report**

In part (c), a large number of candidates were able to simply write down the correct value of \( h \), as intended by the command term in this question. A few candidates wrote down the incorrect negative value. Most candidates attempted to substitute the \( x \) and \( y \) values of the known point correctly into the function, but again many arithmetic and algebraic errors kept them from finding the correct value for \( a \).

Consider the functions \( f(x) \), \( g(x) \) and \( h(x) \). The following table gives some values associated with these functions.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>( 2 )</td>
<td>( 3 )</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>( -14 )</td>
<td>( -18 )</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>( g'(x) )</td>
<td>( -5 )</td>
<td>( -3 )</td>
</tr>
<tr>
<td>( h''(x) )</td>
<td>( -6 )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

14a. Write down the value of \( g(3) \), of \( f'(3) \), and of \( h''(2) \). [3 marks]

**Markscheme**

\( g(3) = -18 \), \( f'(3) = 1 \), \( h''(2) = -6 \quad A1A1A1 \quad N3 \)

[3 marks]

**Examiners report**

Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.
The following diagram shows parts of the graphs of \( h \) and \( h'' \).

There is a point of inflexion on the graph of \( h \) at \( P \), when \( x = 3 \).

Given that \( h(x) = f(x) \times g(x) \),

14b. find the \( y \)-coordinate of \( P \). \[2 \text{ marks}\]

**Markscheme**

writing \( h(3) \) as a product of \( f(3) \) and \( g(3) \) \( A1 \)

\[ f(3) \times g(3) , 3 \times (-18) \]

\( h(3) = -54 \) \( A1 \ N1 \)

\[ 2 \text{ marks} \]

**Examiners report**

Nearly all candidates who attempted to answer parts (a) and (c) did so correctly, as these questions simply required them to understand the notation being used and to read the values from the given table.

The following diagram shows a circle with centre \( O \) and radius \( r \) cm.

![Diagram](image)

Points \( A \) and \( B \) are on the circumference of the circle and \( \angle AOB = 1.4 \) radians.

The point \( C \) is on \( [OA] \) such that \( \angle BCO = \frac{\pi}{3} \) radians.

15. The area of the shaded region is 25 cm\(^2\). Find the value of \( r \). \[7 \text{ marks}\]
**Markscheme**

correct value for BC

\[ BC = rsin\,1.4\cdot \sqrt{r^2 - (r\,cos\,1.4)^2} \quad (AI) \]

area of \( \triangle OBC = \frac{1}{2}r\,(sin\,1.4\times r\,cos\,1.4) = \frac{1}{2}r^2\,sin\,1.4\times cos\,1.4 \quad AI \]

area of sector OAB = \( \frac{1}{2}r^2 \times 1.4 \quad AI \)

attempt to subtract in any order \( (MI) \)

eg sector – triangle, \( \frac{1}{2}r^2\,sin\,1.4\times cos\,1.4 - 0.7r^2 \)

correct equation \( AI \)

eg \( 0.7r^2 - \frac{1}{2}r\,(sin\,1.4\times r\,cos\,1.4) = 25 \)

attempt to solve their equation \( (MI) \)

eg sketch, writing as quadratic, \( \frac{25}{0.606...} \)

\( r = 6.37 \quad AI \quad N4 \)

[7 marks]

Note: Exception to FT rule. Award A1FT for a correct FT answer from a quadratic equation involving two trigonometric functions.

**Examiners report**

As to be expected, candidates found this problem challenging. Those who used a systematic approach in part (b) were more successful than those whose work was scattered about the page. While a pleasing number of candidates successfully found the area of sector AOB, far fewer were able to find the area of triangle BOC. Candidates who took an analytic approach to solving the resulting equation were generally less successful than those who used their GDC. Candidates who converted the angle to degrees generally were not very successful.

---

Let \( f(x) = e^x \) and \( g(x) = mx \), where \( m \geq 0 \), and \(-5 \leq x \leq 5\). Let \( R \) be the region enclosed by the y-axis, the graph of \( f \), and the graph of \( g \).

Let \( m = 1 \).

16a. Sketch the graphs of \( f \) and \( g \) on the same axes. \[ 2 \text{ marks} \]
Notes: Award A1 for the graph of \( f \) positive, increasing and concave up.

Award A1 for graph of \( g \) increasing and linear with \( y \)-intercept of 0.

Penalize one mark if domain is not \([-5, 5] \) and/or if \( f \) and \( g \) do not intersect in the first quadrant.

[2 marks]

Examiners report

There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

While some candidates sketched accurate graphs on the given domain, the majority did not. Besides the common domain error, some exponential curves were graphed with several concavity changes.

16b. Consider all values of \( m \) such that the graphs of \( f \) and \( g \) intersect. Find the value of \( m \) that gives the greatest value for the area [8 marks] of \( R \).
Markscheme
recognize that area of $R$ is a maximum at point of tangency \((R1)\)

$eg\; m = f'(x)$  

equating functions \((MI)\)

$eg\; f(x) = g(x), e^{x} = mx$

$f'(x) = \frac{1}{4}e^{x}$ \((A1)\)

equating gradients \((A1)\)

$eg\; f'(x) = g'(x), \frac{1}{4}e^{x} = m$

attempt to solve system of two equations for $x$ \((MI)\)

$eg\; \frac{1}{4}e^{x} \times x = e^{x}$

$x = 4$ \((A1)\)

attempt to find $m$ \((MI)\)

$eg\; f'(4), \frac{1}{4}e^{4}$

$m = \frac{1}{4}e$ (exact), 0.680 \(AI\ N3\)

[8 marks]

Examiners report
There was a flaw with the domain noted in this question. While not an error in itself, it meant that part (b) no longer assessed what was intended. The markscheme included a variety of solutions based on candidate work seen, and examiners were instructed to notify the IB assessment centre of any candidates adversely affected, and these were looked at during the grade award meeting.

While some candidates were able to show some good reasoning in part (b), fewer were able to find the value of $m$ which maximized the area of the region. In addition to the answer obtained from the restricted domain, full marks were awarded for the answer obtained by using the point of tangency.

Consider the points $A(5, 2, 1), B(6, 5, 3)$, and $C(7, 6, a + 1), a \in \mathbb{R}$.

17a. Find \([3\; marks]\)

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AC}$.

Markscheme

(i) appropriate approach \((MI)\)

$eg\; \overrightarrow{AO} + \overrightarrow{OB}, B - A$

$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ \(AI\ N2\)

(ii) $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 4 \\ a \end{pmatrix}$ \(AI\ N1\)

[3 marks]
17b. Find the value of $a$ for which $q = \frac{\pi}{2}$. [4 marks]

Markscheme
valid reasoning (seen anywhere) $\textbf{R1}$
$eg$ scalar product is zero, $\cos \frac{\pi}{2} = \frac{\mathbf{AB} \cdot \mathbf{AC}}{|\mathbf{AB}| |\mathbf{AC}|}$
correct scalar product of their $\mathbf{AB}$ and $\mathbf{AC}$ (may be seen in part (c)) $\textbf{(AI)}$
$eg$ $1(2) + 3(4) + 2(a)$
correct working for their $\mathbf{AB}$ and $\mathbf{AC}$ $\textbf{(AI)}$
$eg$ $2a + 14, 2a = -14$
$a = -7$ $\textbf{AI}$ $\textbf{N3}$
[4 marks]

Examiners report
In part (b), while most candidates correctly found the value of $a$, many unnecessarily worked with the magnitudes of the vectors, sometimes leading to algebra errors.

17c. Show that $\cos q = \frac{2a + 14}{\sqrt{14a^2 + 280}}$. [4 marks]

Markscheme
correct magnitudes (may be seen in (b)) $\textbf{(AI)(AI)}$
$\sqrt{1^2 + 3^2 + 2^2} \left( = \sqrt{14} \right) \cdot \sqrt{2^2 + 4^2 + a^2} \left( = \sqrt{20 + a^2} \right)$
substitution into formula $\textbf{(M1)}$
$eg$ $\cos \theta = \frac{1 + 2 + 3 + 4 + 2a}{\sqrt{1^2 + 3^2 + 2^2} \sqrt{2^2 + 4^2 + a^2}} = \frac{14 + 2a}{\sqrt{20 + a^2}}$
simplification leading to required answer $\textbf{AI}$
$eg$ $\cos \theta = \frac{14 + 2a}{\sqrt{20 + a^2}}$
$\cos \theta = \frac{2a + 14}{\sqrt{14a^2 + 280}} \textbf{AG} \textbf{ N0}$
[4 marks]

Examiners report
Some candidates showed a minimum of working in part (c)(i); in a “show that” question, candidates need to ensure that their working clearly leads to the answer given. A common error was simplifying the magnitude of vector $\mathbf{AC}$ to $\sqrt{20a^2}$ instead of $\sqrt{20 + a^2}$. 

Examiners report
The majority of candidates successfully found the vectors between the given points in part (a).
Consider the points $A(5, 2, 1)$, $B(6, 5, 3)$, and $C(7, 6, a+1)$, $a \in \mathbb{R}$.

Let $q$ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

17d. Hence, find the value of $a$ for which $q = 1.2$. [4 marks]

### Markscheme

**Correct setup**  \((A1)\)

**eg** \(\cos 1.2 = \frac{2a+14}{\sqrt{160a+280}}\)

**Valid attempt to solve**  \((M1)\)

**eg** sketch, \(\frac{2a+14}{\sqrt{160a+280}} - \cos 1.2 = 0\), attempt to square

\(a = -3.25\)  \(A2\)  \(N3\)  [4 marks]

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### Examiners report

In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

---

A particle’s displacement, in metres, is given by \(s(t) = 2t \cos t\), for \(0 \leq t \leq 6\), where \(t\) is the time in seconds.

18a. On the grid below, sketch the graph of \(s\). [4 marks]
**Markscheme**

Note: Award A1 for approximately correct shape (do not accept line segments).

Only if this A1 is awarded, award the following:
- A1 for maximum and minimum within circles,
- A1 for x-intercepts between 1 and 2 and between 4 and 5,
- A1 for left endpoint at (0, 0) and right endpoint within circle.

[4 marks]

**Examiners report**

Most candidates sketched an approximately correct shape for the displacement of a particle in the given domain, but many lost marks for carelessness in graphing the local extrema or the right endpoint.

18b. Find the maximum velocity of the particle. [3 marks]

**Markscheme**

appropriate approach (MI)
- e.g. recognizing that \( v = s' \), finding derivative, \( a = s'' \)
- valid method to find maximum (MI)
- e.g. sketch of \( v, v'(t) = 0, t = 5.08698 \ldots \)
- \( v = 10.20025 \ldots \)
- \( v = 10.2 \) [10.2, 10.3] A1 N2

[3 marks]

**Examiners report**

In part (b), most candidates knew to differentiate displacement to find velocity, but few knew how to then find the maximum. Occasionally, a candidate would give the time value of the maximum. Others attempted to incorrectly set the first derivative equal to zero and solve analytically rather than take the maximum value from the graph of the velocity function.
Consider the function \( f(x) = x^2 - 4x + 1 \).

19a. Sketch the graph of \( f \), for \(-1 \leq x \leq 5\). [4 marks]

**Markscheme**

**Note:** The shape **must** be an approximately correct upwards parabola.

**Only** if the shape is approximately correct, award the following:

\( A1 \) for vertex \( x \approx 2 \), \( A1 \) for \( x \)-intercepts between 0 and 1, and 3 and 4, \( A1 \) for correct \( y \)-intercept (0, 1), \( A1 \) for correct domain \([-1, 5]\).

Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

**Examiners report**

A good number of students provided a clear sketch of the quadratic function within the given domain. Some lost marks as they did not clearly indicate the approximate positions of the most important points of the parabola either by labelling or providing a suitable scale.

19b. This function can also be written as \( f(x) = (x - p)^2 - 3 \). [1 mark]

Write down the value of \( p \).

**Markscheme**

\( p = 2 \) \( A1 \) \( N1 \)

[1 mark]

**Examiners report**

There were few difficulties in part (b).
19c. The graph of $g$ is obtained by reflecting the graph of $f$ in the $x$-axis, followed by a translation of \( \begin{pmatrix} 0 \\ 6 \end{pmatrix} \).

Show that $g(x) = -x^2 + 4x + 5$.

**Markscheme**

correct vertical reflection, correct vertical translation \( (A1)(A1) \)

e.g. $f(x), -(x - 2)^2 - 3, -y, -f(x) + 6, y + 6$

transformations in correct order \( (A1) \)

e.g. $-x^2 - 4x + 1 + 6, -(x - 2)^2 + 3 + 6$

simplification which clearly leads to given answer \( A1 \)

e.g. $-x^2 + 4x - 1 + 6, -(x^2 - 4x + 4 - 3) + 6$

$g(x) = -x^2 + 4x + 5 \quad AG \quad N0$

Note: If working shown, award $A1A1A0A0$ if transformations correct, but done in reverse order, e.g. $-(x^2 - 4x + 1 + 6)$.

[4 marks]

**Examiners report**

In part (c), candidates often used an insufficient number of steps to show the required result or had difficulty setting out their work logically.

19d. The graph of $g$ is obtained by reflecting the graph of $f$ in the $x$-axis, followed by a translation of \( \begin{pmatrix} 0 \\ 6 \end{pmatrix} \).

The graphs of $f$ and $g$ intersect at two points.

Write down the $x$-coordinates of these two points.

**Markscheme**

valid approach \( (M1) \)

e.g. sketch, $f = g$

$-0.449489\ldots, 4.449489\ldots$

$(2 \pm \sqrt{3})$ (exact), $-0.449 \pm [0.450, -0.449]; 4.44 \pm [4.44, 4.45] \quad A1A1 \quad N3$

[3 marks]

**Examiners report**

Part (d) was generally done well though many candidates gave at least one answer to fewer than three significant figures, potentially resulting in more lost marks.

19e. The graph of $g$ is obtained by reflecting the graph of $f$ in the $x$-axis, followed by a translation of \( \begin{pmatrix} 0 \\ 6 \end{pmatrix} \).

Let $R$ be the region enclosed by the graphs of $f$ and $g$.

Find the area of $R$.
**Markscheme**

attempt to substitute limits or functions into area formula (accept absence of $dx$)  \((M1)\)

\[ \int_a^b \left( (-x^2 + 4x + 5) - (x^2 - 4x + 1) \right) dx \cdot \int_{-0.449}^{0.45} (f - g) \cdot \int_{-2}^{8} (2x^2 + 8x + 4) dx \]

approach involving subtraction of integrals/areas (accept absence of $dx$)  \((M1)\)

\[ \int_0^b (-x^2 + 4x + 5) - \int_0^b (x^2 - 4x + 1) \cdot \int (f - g) dx \]

area = 39.19183...

area = 39.2 [39.1, 39.2] \(A1\) \(N3\)

[3 marks]

**Examiners report**

In part (e), many candidates were unable to connect the points of intersection found in part (d) with the limits of integration. Mistakes were also made here either using a GDC incorrectly or not subtracting the correct functions. Other candidates tried to divide the region into four areas and made obvious errors in the process. Very few candidates subtracted \(\int_a^b\) to get a simple function before integrating and there were numerous, fruitless analytical attempts to find the required integral.

Let \(f(x) = 2x - 1\) and \(g(x) = 3x^2 + 2\).

20a. Find \(f^{-1}(x)\).  \([3\text{ marks}]\)

**Markscheme**

interchanging \(x\) and \(y\) (seen anywhere)  \((M1)\)

\[ e.g. x = 2y - 1 \]

correct manipulation  \((AI)\)

\[ e.g. x + 1 = 2y \]

\[ f^{-1}(x) = \frac{x + 1}{2} \] \(A1\) \(N2\)

[3 marks]

**Examiners report**

This question was answered correctly by nearly all candidates.

20b. Find \((f \circ g)(1)\).  \([3\text{ marks}]\)
**Markscheme**

**METHOD 1**

attempt to find or \( g(1) \) or \( f(1) \) \((M1)\)

\[ g(1) = 5 \] \((A1)\)

\[ f(5) = 9 \] \(A1\) \(N2\)

\([3\text{ marks}]\)

**METHOD 2**

attempt to form composite (in any order) \((M1)\)

e.g. \(2(3x^2 + 2) - 1\), \(3(2x - 1)^2 + 2\)

\[ (f \circ g)(1) = 2(3 \times 1^2 + 2) - 1 = 6 \times 1^2 + 3 \] \((A1)\)

\[ (f \circ g)(1) = 9 \] \(A1\) \(N2\)

\([3\text{ marks}]\)

**Examiners report**

This question was answered correctly by nearly all candidates. In part (b), there were a few who seemed unfamiliar with the notation for composition of functions, and attempted to multiply the functions rather than finding the composite, and there were a few who found the correct composite function but failed to substitute in \(x = 1\) to find the value.
The diagram below shows the graph of a function $f(x)$, for $-2 \leq x \leq 3$.

21a. Sketch the graph of $f(-x)$ on the grid below. [2 marks]
Examiners report

In part (a) of this question, a large number of candidates correctly sketched the graph of \( f(-x) \), as asked. A fairly common error, however, was to graph \(-f(x)\).

21b. The graph of \( f \) is transformed to obtain the graph of \( g \). The graph of \( g \) is shown below. 

The function \( g \) can be written in the form \( g(x) = af(x + b) \). Write down the value of \( a \) and of \( b \).
22. Consider the equation $x^2 + (k - 1)x + 1 = 0$, where $k$ is a real number. Find the values of $k$ for which the equation has two equal real solutions. [7 marks]

Markscheme

**METHOD 1**

evidence of valid approach (M1)
e.g. $b^2 - 4ac$, quadratic formula
correct substitution into $b^2 - 4ac$ (may be seen in formula) (AI)
e.g. $(k - 1)^2 - 4 \times 1 \times 1$, $(k - 1)^2 - 4$, $k^2 - 2k - 3$
setting their discriminant equal to zero MI
e.g. $\Delta = 0, (k - 1)^2 - 4 = 0$
attempt to solve the quadratic (M1)
e.g. $(k - 1)^2 = 4$, factorizing
correct working A1
e.g. $(k - 1) = \pm 2, (k - 3)(k + 1)$
$k = -1, k = 3$ (do not accept inequalities) A1A1 N2

**METHOD 2**

recognizing perfect square (M1)
e.g. $(x + 1)^2 = 0, (x - 1)^2$
correct expansion (AI)(AI)
e.g. $x^2 + 2x + 1 = 0, x^2 - 2x + 1$
equating coefficients of $x$ A1A1
e.g. $k - 1 = -2, k - 1 = 2$
$k = -1, k = 3$ A1A1 N2

Examiners report

Most candidates approached this question correctly by using the discriminant, and many were successful in finding both of the required values of $k$. There did seem to be some confusion about the expression "two equal real solutions", as some candidates approached the question as though the equation had two distinct real roots, using $b^2 - 4ac > 0$, rather than $b^2 - 4ac = 0$.

There were also a good number who recognized that the quadratic must be a perfect square, although many who used this method found only one of the two possible values of $k$. In addition, there were many unsuccessful candidates who tried to use the entire quadratic formula as though they were solving for $x$, without ever seeming to realize the significance of the discriminant.
The following diagram shows the graph of a quadratic function $f$, for $0 \leq x \leq 4$.

The graph passes through the point $P(0, 13)$, and its vertex is the point $V(2, 1)$.

23a. The function can be written in the form $f(x) = a(x - h)^2 + k$.

(i) Write down the value of $h$ and of $k$.

(ii) Show that $a = 3$.

**Markscheme**

(i) $h = 2, k = 1$ \( \text{A1A1 N2} \)

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into $f$ \( \text{M1} \)

e.g. $13 = a(0 - 2)^2 + 1$

working towards solution \( \text{A1} \)

e.g. $13 = 4a + 1$

$a = 3 \text{ AG N0} \quad \text{[4 marks]}$

**Examiners report**

In part (a), nearly all the candidates recognized that $h$ and $k$ were the coordinates of the vertex of the parabola, and most were able to successfully show that $a = 3$. Unfortunately, a few candidates did not understand the "show that" command, and simply verified that $a = 3$ would work, rather than showing how to find $a = 3$.

23b. Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$.

\[
f(x) = 3(x^2 - 2x + 4) + 1 \quad (x - 2)^2 = x^2 - 4x + 4
\]

\[
f(x) = 3x^2 - 12x + 12 + 1
\]

\[
f(x) = 3x^2 - 12x + 13 \quad A = 3 \quad B = -12 \quad C = 13
\]
Examiners report
In part (b), most candidates were able to find \( f(x) \) in the required form. For a few candidates, algebraic errors kept them from finding the correct function, even though they started with correct values for \( a, h \) and \( k \).

23c. Calculate the area enclosed by the graph of \( f \), the \( x \)-axis, and the lines \( x = 2 \) and \( x = 4 \). [8 marks]
**Markscheme**

**METHOD 1**

integral expression \( (A1) \)
e.g. \( \int_2^4 (3x^2 - 12x + 13) \, f'\!dx \)

\[
\text{Area} = \left[ x^3 - 6x^2 + 13x \right]_2^4 \quad A1A1A1
\]

*Note:* Award \( A1 \) for \( x^3 \), \( A1 \) for \(-6x^2\), \( A1 \) for \( 13x \).

correct substitution of \( \textit{correct} \) limits into \( \textit{their} \) expression \( A1A1 \)
e.g. \( (4^3 - 6 \times 4^2 + 13 \times 4) - (2^3 - 6 \times 2^2 + 13 \times 2) \), \( 64 - 96 + 52 - (8 - 24 + 26) \)

*Note:* Award \( A1 \) for substituting 4, \( A1 \) for substituting 2.

correct working \( (A1) \)
e.g. \( 64 - 96 + 52 - 8 + 24 - 26, 20 - 10 \)

\[
\text{Area} = 10 \quad A1 \quad N3
\]

\[8 \text{ marks}\]

**METHOD 2**

integral expression \( (A1) \)
e.g. \( \int_2^4 (3(x - 2)^2 + 1) \, f'\!dx \)

\[
\text{Area} = \left[ (x - 2)^3 + x \right]_2^4 \quad A2A1
\]

*Note:* Award \( A2 \) for \( (x - 2)^3 \), \( A1 \) for \( x \).

correct substitution of \( \textit{correct} \) limits into \( \textit{their} \) expression \( A1A1 \)
e.g. \( (4 - 2)^3 + 4 - [(2 - 2)^3 + 2] \), \( 2^3 + 4 - (0^3 + 2) \), \( 2^3 + 4 - 2 \)

*Note:* Award \( A1 \) for substituting 4, \( A1 \) for substituting 2.

correct working \( (A1) \)
e.g. \( 8 + 4 - 2 \)

\[
\text{Area} = 10 \quad A1 \quad N3
\]

\[8 \text{ marks}\]

**METHOD 3**

recognizing area from 0 to 2 is same as area from 2 to 4 \( (R1) \)
e.g. sketch, \( \int_2^4 f'\!dx = \int_0^2 f'\!dx \)

integral expression \( (A1) \)
e.g. \( \int_0^2 (3x^2 - 12x + 13) \, f'\!dx \)

\[
\text{Area} = \left[ x^3 - 6x^2 + 13x \right]_0^2 \quad A1A1A1
\]

*Note:* Award \( A1 \) for \( x^3 \), \( A1 \) for \(-6x^2\), \( A1 \) for \( 13x \).

correct substitution of \( \textit{correct} \) limits into \( \textit{their} \) expression \( A1(A1) \)
e.g. \( (2^3 - 6 \times 2^2 + 13 \times 2) - (0^3 - 6 \times 0^2 + 13 \times 0) \), \( 8 - 24 + 26 \)

*Note:* Award \( A1 \) for substituting 2, \( A1(A1) \) for substituting 0.

\[
\text{Area} = 10 \quad A1 \quad N3
\]

\[8 \text{ marks}\]
Examiners report
In part (c), nearly all candidates knew that they needed to integrate to find the area, but errors in integration, and algebraic and arithmetic errors prevented many from finding the correct area.

Let \( f(x) = \frac{x}{2x^2+5x-2} \) for \(-2 \leq x \leq 4, \ x \neq \frac{1}{2}, \ x \neq 2\). The graph of \( f \) is given below.

The graph of \( f \) has a local minimum at \( A(1, 1) \) and a local maximum at \( B \).

24a. Use the quotient rule to show that \( f'(x) = \frac{2x-2}{(-2x^2+5x-2)^2} \). [6 marks]

Markscheme
Correct derivatives applied in quotient rule \((AI)\) \( A1 \)

1. \(-4x+5\)

Note: Award \((AI)\) for 1, \(AI\) for \(-4x\) and \(AI\) for 5, only if it is clear candidates are using the quotient rule.

Correct substitution into quotient rule \( AI \)

e.g. \( \frac{1 \cdot (-2x^2+5x-2) - x(-4x+5)}{(-2x^2+5x-2)^2} \), \( \frac{-2x^2+5x-2(-4x+5)}{(-2x^2+5x-2)^2} \)

Correct working \((AI)\)

e.g. \( \frac{-2x^2+5x-2(-4x+5)}{(-2x^2+5x-2)^2} \)

Expression clearly leading to the answer \( AI \)

e.g. \( \frac{-2x^2+5x-2+4x^2+5x}{(-2x^2+5x-2)^2} \)

\( f'(x) = \frac{2x-2}{(-2x^2+5x-2)^2} \) \( AG \) \( N0 \)

[6 marks]

Examiners report
While most candidates answered part (a) correctly, there were some who did not show quite enough work for a "show that" question. A very small number of candidates did not follow the instruction to use the quotient rule.
24b. Hence find the coordinates of B.

**Markscheme**

evidence of attempting to solve \( f'(x) = 0 \)  \((M1)\)
e.g. \( 2x^2 - 2 = 0 \)
evidence of correct working  \((AI)\)
e.g. \( x^2 = 1, \frac{\pm \sqrt{3}}{2}, 2(x-1)(x+1) \)
correct solution to quadratic  \((AI)\)
e.g. \( x = \pm 1 \)
correct \( x \)-coordinate \( x = -1 \) (may be seen in coordinate form \( (-1, \frac{1}{9}) \) )  \((AI)\)  \((N2)\)

**Examiners report**

In part (b), most candidates knew that they needed to solve the equation \( f'(x) = 0 \), and many were successful in answering this question correctly. However, some candidates failed to find both values of \( x \), or made other algebraic errors in their solutions. One common error was to find only one solution for \( x^2 - 1 \); another was to work with the denominator equal to zero, rather than the numerator.

24c. Given that the line \( y = k \) does not meet the graph of \( f \), find the possible values of \( k \).

**Markscheme**

recognizing values between max and min  \((R1)\)

\( \frac{1}{9} < k < 1 \)  \((A2)\)  \((N3)\)

**Examiners report**

In part (c), a significant number of candidates seemed to think that the line \( y = k \) was a vertical line, and attempted to find the vertical asymptotes. Others tried looking for a horizontal asymptote. Fortunately, there were still a good number of intuitive candidates who recognized the link with the graph and with part (b), and realized that the horizontal line must pass through the space between the given local minimum and the local maximum they had found in part (b).
Let \( f(x) = \log_p(x + 3) \) for \( x > -3 \). Part of the graph of \( f \) is shown below.

The graph passes through \( A(6, 2) \), has an \( x \)-intercept at \((-2, 0)\) and has an asymptote at \( x = -3 \).

25a. Find \( p \). \[4 \text{ marks}\]

**Markscheme**

- evidence of substituting the point \( A \) \( (M1) \)
- e.g. \( 2 = \log_p(6 + 3) \)
- manipulating logs \( A1 \)
- e.g. \( p^2 = 9 \)
- \( p = 3 \) \( A2 \) \( N2 \)

\[4 \text{ marks}\]

**Examiners report**

In part (a), many candidates successfully substituted the point \( A \) to find the base of the logarithm, although some candidates lost a mark for not showing their manipulation of the logarithm equation into the exponential equation.

25b. The graph of \( f \) is reflected in the line \( y = x \) to give the graph of \( g \). \[5 \text{ marks}\]

(i) Write down the \( y \)-intercept of the graph of \( g \).

(ii) Sketch the graph of \( g \), noting clearly any asymptotes and the image of \( A \).
**Markscheme**

(i) \( y = -2 \) (accept \((0, -2)\))  \( A1 \ N1 \)

(ii)

**Note:** Award \( A1 \) for asymptote at \( y = -3 \), \( A1 \) for an increasing function that is concave up, \( A1 \) for a positive \( x \)-intercept and a negative \( y \)-intercept, \( A1 \) for passing through the point \((2, 6)\).

\[ 5 \text{ marks} \]

**Examiners report**

A number of candidates who correctly stated the \( y \)-intercept was \(-2\) had difficulty sketching the graph of the reflection in the line \( y = x \). A number of candidates graphed directly on the question paper rather than sketching their own graph; candidates should be reminded to show all working for Section B on separate paper. Some correct sketches did not have the position of A indicated. Many candidates had difficulty reflecting the asymptote.

25c. The graph of \( f \) is reflected in the line \( y = x \) to give the graph of \( g \).

Find \( g(x) \).  \[ 4 \text{ marks} \]
**Markscheme**

**METHOD 1**
recognizing that \( g = f^{-1} \) \((R1)\)
evidence of valid approach \( (M1)\)
e.g. switching \( x \) and \( y \) (seen anywhere), solving for \( x \)
correct manipulation \( (A1)\)
e.g. \( 3^x = y + 3 \)
\( g(x) = 3^x - 3 \) \( A1 \) \( N3 \)

**METHOD 2**
recognizing that \( g(x) = a^x + b \) \((R1)\)
identifying vertical translation \( (A1)\)
e.g. graph shifted down 3 units, \( f(x) - 3 \)
evidence of valid approach \( (M1)\)
e.g. substituting point to identify the base
\( g(x) = 3^x - 3 \) \( A1 \) \( N3 \)

**[4 marks]**

**Examiners report**
Part (c) was often well done, with candidates showing clear and correct working.
The most successful candidates clearly appreciated the linkage between the question parts.

---

Let \( f(x) = 2x^2 - 8x - 9 \).

26a. (i) Write down the coordinates of the vertex. \([4 \text{ marks}]\)

(ii) Hence or otherwise, express the function in the form \( f(x) = 2(x - h)^2 + k \).

**Markscheme**

(i) \((2, -17)\) or \( x = 2 , y = -17 \) \( A1A1 \) \( N2 \)

(ii) evidence of valid approach \( (M1)\)
e.g. graph, completing the square, equating coefficients
\( f(x) = 2(x - 2)^2 - 17 \) \( A1 \) \( N2 \)

**[4 marks]**

**Examiners report**
This question was well done by the majority of candidates.

26b. Solve the equation \( f(x) = 0 \). \([3 \text{ marks}]\)
Markscheme

evidence of valid approach \hspace{0.5cm} (M1)

e.g. graph, quadratic formula

\[-0.915475\ldots, 4.915475\ldots\]

\[x = -0.915, 4.92\] \hspace{0.5cm} A1A1 \hspace{0.5cm} N3

[3 marks]

Examiners report

This question was well done by the majority of candidates. There were still many however who opted for an analytical approach in part (b), which often led to errors in sign and accuracy. Some candidates used the trace feature on their GDC to find the vertex which often resulted in accuracy errors.

\[
y = (x - 1)\sin x, \text{ for } 0 \leq x \leq \frac{5\pi}{2}, \text{ is shown below.}
\]

The graph has \(x\)-intercepts at 0, 1, \(\pi\) and \(k\).

27a. Find \(k\). \hspace{1cm} [2 marks]

Markscheme

evidence of valid approach \hspace{0.5cm} (M1)

e.g. \(y = 0, \sin x = 0\)

\(2\pi = 6.283185\ldots\)

\(k = 6.28\) \hspace{0.5cm} A1 \hspace{0.5cm} N2

[2 marks]

Examiners report

Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of \(dx\), using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of \(\pi\). There were still many who were unable to use their calculator successfully to find the required volume.

27b. The shaded region is rotated 360° about the \(x\)-axis. Let \(V\) be the volume of the solid formed. \hspace{1cm} [3 marks]

Write down an expression for \(V\).
Markscheme

attempt to substitute either limits or the function into formula \((M1)\)  
(accept absence of \(dx\))  
e.g. \(V = \pi \int_{a}^{b} (f(x))^2 \, dx, \pi \int ((x - 1) \sin x)^2, \pi \int_{a}^{b} y^2 \, dx\)  
correct expression \(A2 \quad N3\)  
e.g. \(\pi \int_{a}^{b} (x - 1)^2 \sin^2 x \, dx, \pi \int_{a}^{b} ((x - 1) \sin x)^2 \, dx\)  
[3 marks]

Examiners report

Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of \(dx\), using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of \(\pi\). There were still many who were unable to use their calculator successfully to find the required volume.

27c. The shaded region is rotated \(360^\circ\) about the \(x\)-axis. Let \(V\) be the volume of the solid formed. \([2 \text{ marks}]\)

Find \(V\).

Markscheme

\(V = 69.60192562\ldots\)  
\(V = 69.6\quad A2 \quad N2\)  
[2 marks]

Examiners report

Candidates showed marked improvement in writing fully correct expressions for a volume of revolution. Common errors of course included the omission of \(dx\), using the given domain as the upper and lower bounds of integration, forgetting to square their function and/or the omission of \(\pi\). There were still many who were unable to use their calculator successfully to find the required volume.

The following diagram shows two ships A and B. At noon, ship A was 15 km due north of ship B. Ship A was moving south at 15 km \(h^{-1}\) and ship B was moving east at 11 km \(h^{-1}\).

28a. Find the distance between the ships \([5 \text{ marks}]\)

(i) at 13:00;  
(ii) at 14:00.
Markscheme
(i) evidence of valid approach \( (M1) \)
e.g. ship A where B was, B away \( \text{A1 N2} \)
(ii) evidence of valid approach \( (M1) \)
e.g. new diagram, Pythagoras, vectors
\[
s = \sqrt{15^2 + 22^2} \quad (A1)
\]
\[
\sqrt{709} = 26.62705
\]
\[s = 26.6 \quad \text{A1 N2}
\]
Note: Award \( M0A0A0 \) for using the formula given in part (b).
[5 marks]

Examiners report
Part (a) was generally well done although some candidates incorrectly used the function given in part (b) to find the required values. There was evidence that some candidates are not comfortable with a 24-hour clock.

28b. Let \( s(t) \) be the distance between the ships \( t \) hours after noon, for \( 0 \leq t \leq 4 \).

Show that \( s(t) = \sqrt{346t^2 - 450t + 225} \).

Markscheme
evidence of valid approach \( (M1) \)
e.g. a table, diagram, formula \( d = r \times t \)
distance ship A travels \( t \) hours after noon is \( 15(t - 1) \) \( (A2) \)
distance ship B travels in \( t \) hours after noon is \( 11t \) \( (A1) \)
evidence of valid approach \( M1 \)
e.g. \( s(t) = \sqrt{[15(t - 1)]^2 + (11t)^2} \)
correct simplification \( A1 \)
e.g. \( \sqrt{225(t^2 - 2t + 1) + 121t^2} \)
\[s(t) = \sqrt{346t^2 - 450t + 225} \quad AG \ N0
\]
[6 marks]

Examiners report
Candidates had difficulty generalizing the problem and therefore, were unable to show how the function \( s(t) \) was obtained in part (b).

28c. Sketch the graph of \( s(t) \).

[3 marks]
**Markscheme**

Note: Award A1 for shape, A1 for minimum at approximately (0.7, 9), A1 for domain.

[3 marks]

**Examiners report**

Surprisingly, the graph in part (c) was not well done. Candidates often ignored the given domain, provided no indication of scale, and drew "V" shapes or parabolas.

28d. Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00.

**Markscheme**

evidence of valid approach (M1)

e.g. \( s'(t) = 0 \), find minimum of \( s(t) \), graph, reference to "more than 8 km"

\[ \text{min} = 8.870455 \ldots \text{(accept 2 or more sf)} \]  A1

since \( s_{\text{min}} > 8 \), captain cannot see ship B  R1  N0

[3 marks]

**Examiners report**

In part (d), candidates simply regurgitated the question without providing any significant evidence for their statements that the two ships must have been more than 8 km apart.

Let \( f(x) = \cos(e^x) \), for \(-2 \leq x \leq 2\).

29a. Find \( f'(x) \).  [2 marks]
Markscheme

\[ f'(x) = -e^x \sin(e^x) \] A1 A1 N2 [2 marks]

Examiners report

Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

29b. On the grid below, sketch the graph of \( f'(x) \). [4 marks]
**Markscheme**

Note: Award A1 for shape that must have the correct domain (from $-2$ to $+2$) and correct range (from $-6$ to $4$), A1 for minimum in circle, A1 for maximum in circle and A1 for intercepts in circles.

[4 marks]

**Examiners report**

Many students failed in applying the chain rule to find the correct derivative, and some inappropriately used the product rule. However, many of those obtained full follow through marks in part (b) for the sketch of the function they found in part (a).

Most candidates sketched an approximately correct shape in the given domain, though there were some that did not realize they had to set their GDC to radians, producing a meaningless sketch.

It is very important to stress to students that although they are asked to produce a sketch, it is still necessary to show its key features such as domain and range, stationary points and intercepts.

Let $f(x) = ax^3 + bx^2 + c$ , where $a$ , $b$ and $c$ are real numbers. The graph of $f$ passes through the point $(2, 9)$.

30a. Show that $8a + 4b + c = 9$.

**Markscheme**

attempt to substitute coordinates in $f$ (M1)

e.g. $f(2) = 9$

correct substitution A1

e.g. $a \times 2^3 + b \times 2^2 + c = 9$

$8a + 4b + c = 9$ AG N0

[2 marks]
Examiners report
Part (a) was generally well done, with a few candidates failing to show a detailed substitution. Some substituted 2 in place of $x$, but didn’t make it clear that they had substituted in $y$ as well.

30b. The graph of $f$ has a local minimum at $(1, 4)$. [7 marks]

Find two other equations in $a$, $b$, and $c$, giving your answers in a similar form to part (a).

Markscheme
recognizing that $(1, 4)$ is on the graph of $f$ \((MI)\)
e.g. $f(1) = 4$
correct equation \(A1\)
e.g. $a + b + c = 4$
recognizing that $f’ = 0$ at minimum (seen anywhere) \((MI)\)
e.g. $f’(1) = 0$
$f’(x) = 3ax^2 + 2bx$ (seen anywhere) \(A1A1\)
correct substitution into derivative \((AI)\)
e.g. $3a \times 1^2 + 2b \times 1 = 0$
correct simplified equation \(A1\)
e.g. $3a + 2b = 0$

Examiners report
A great majority could find the two equations in part (b). However there were a significant number of candidates who failed to identify that the gradient of the tangent is zero at a minimum point, thus getting the incorrect equation $3a + 2b = 4$.

30c. Find the value of $a$, of $b$ and of $c$. [4 marks]

Markscheme
valid method for solving system of equations \((MI)\)
e.g. inverse of a matrix, substitution
\[a = 2, \ b = -3, \ c = 5\ \ A1A1A1 \ N4\]

Examiners report
A considerable number of candidates only had 2 equations, so that they either had a hard time trying to come up with a third equation (incorrectly combining some of the information given in the question) to solve part (c) or they completely failed to solve it.

Despite obtaining three correct equations many used long elimination methods that caused algebraic errors. Pages of calculations leading nowhere were seen.

Those who used matrix methods were almost completely successful.
Consider the following circle with centre O and radius r.

The points P, R and Q are on the circumference, \( PÔQ = 2\theta \), for \( 0 < \theta < \frac{\pi}{2} \).

31a. Use the cosine rule to show that \( PQ = 2r \sin \theta \).

**Markscheme**

- correct substitution into cosine rule \( A1 \)
  
  - e.g. \( PQ^2 = r^2 + r^2 - 2(r)(r) \cos(2\theta) \), \( PQ^2 = 2r^2 - 2r^2 (\cos(2\theta)) \)
  
  - substituting \( 1 - 2\sin^2 \theta \) for \( \cos 2\theta \) (seen anywhere) \( A1 \)
  
  - e.g. \( PQ^2 = 2r^2 - 2r^2 (1 - 2\sin^2 \theta) \)
  
  - working towards answer \( (A1) \)
  
  - e.g. \( PQ^2 = 2r^2 - 2r^2 + 4r^2 \sin^2 \theta \)
  
  - recognizing \( 2r^2 - 2r^2 = 0 \) (including crossing out) (seen anywhere)
  
  - e.g. \( PQ^2 = 4r^2 \sin^2 \theta \), \( PQ = \sqrt{4r^2 \sin^2 \theta} \)

\( PQ = 2r \sin \theta \) \( AG \) \( N0 \)

[4 marks]

**Examiners report**

This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

Part (a) was generally attempted using the cosine rule, but many failed to substitute correctly into the right hand side or skipped important steps. A high percentage could not arrive at the given expression due to a lack of knowledge of trigonometric identities or making algebraic errors, and tried to force their way to the given answer.

The most common errors included taking the square root too soon, and sign errors when distributing the negative after substituting \( \cos 2\theta \) by \( 1 - 2\sin^2 \theta \).

31b. Let \( l \) be the length of the arc PRQ.

Given that \( 1.3PQ - l = 0 \), find the value of \( \theta \).
Markscheme

PRQ = r × 2θ (seen anywhere) \( (AI) \)
correct set up \( AI \)
e.g. \( 1.3 \times 2r \sin \theta - r \times (2\theta) = 0 \)

attempt to eliminate \( r \) \( (MI) \)
correct equation in terms of the one variable \( \theta \) \( (AI) \)
e.g. \( 1.3 \times 2 \sin \theta - 2\theta = 0 \)

1.221496215

\( \theta = 1.22 \) (accept 70.0° (69.9)) \( AI \) \( N3 \)

[5 marks]

Examiners report

This exercise seemed to be challenging for the great majority of the candidates, in particular parts (b), (c) and (d).

In part (b), most candidates understood what was required but could not find the correct length of the arc PRQ mainly due to substituting the angle by \( \theta \) instead of \( 2\theta \).

31c. Consider the function \( f(\theta) = 2.6 \sin \theta - 2\theta \), for \( 0 < \theta < \frac{\pi}{2} \). \( [4 \text{ marks}] \)

(i) Sketch the graph of \( f \).

(ii) Write down the root of \( f(\theta) = 0 \).

Markscheme

(i)

\[
\begin{array}{c}
\text{y}
\end{array}
\]

\( AI\!AIAI N3 \)

Note: Award \( AI \) for approximately correct shape, \( AI \) for \( x \)-intercept in approximately correct position, \( AI \) for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

\( \theta = 1.22 \) \( AI \) \( N1 \)

[4 marks]
Examiners report

Regarding part (c), many valid approaches were seen for the graph of \( f \), making a good use of their GDC. A common error was finding a second or third solution outside the domain. A considerable amount of sketches were missing a scale.

There were candidates who achieved the correct equation but failed to realize they could use their GDC to solve it.

31d. Use the graph of \( f \) to find the values of \( \theta \) for which \( l < 1.3PQ \).

[3 marks]

Markscheme

evidence of appropriate approach (may be seen earlier) \( M2 \)

e.g. \( 2\theta < 2.6\sin \theta, 0 < f(\theta) \), showing positive part of sketch

\( 0 < \theta < 1.221496215 \)

\( 0 < \theta = 1.22 \) (accept \( \theta < 1.22 \)) \( A1 \) \( NI \)

[3 marks]

Examiners report

Part (d) was attempted by very few, and of those who achieved the correct answer not many were able to show the method they used.

Let \( f \) be a quadratic function. Part of the graph of \( f \) is shown below.

The vertex is at \( P(4, 2) \) and the y-intercept is at \( Q(0, 6) \).

32a. Write down the equation of the axis of symmetry.

[1 mark]

Markscheme

\( x = 4 \) (must be an equation) \( A1 \) \( NI \)

[1 mark]

Examiners report

A surprising number of candidates missed part (a) of this question, which required them to write the equation of the axis of symmetry. Some candidates did not write their answer as an equation, while others simply wrote the formula \( x = -\frac{b}{2a} \).

32b. The function \( f \) can be written in the form \( f(x) = a(x - h)^2 + k \).

Write down the value of \( h \) and of \( k \).

[2 marks]
32c. The function $f$ can be written in the form $f(x) = a(x - h)^2 + k$.

Find $a$.

**Markscheme**

attempt to substitute coordinates of any point on the graph into $f$  \( \text{\textit{(MI)}} \)

e.g. $f(0) = 6$, $6 = a(0 - 4)^2 + 2$, $f(4) = 2$

correct equation (do not accept an equation that results from $f(4) = 2$)  \( \text{\textit{(AI)}} \)

e.g. $6 = a(-4)^2 + 2$, $6 = 16a + 2$

$a = \frac{4}{16} \left(= \frac{1}{4} \right)$  \( A1 \)  \( N2 \)

[3 marks]

**Examiners report**

The rest of this question was answered correctly by the large majority of candidates. The mistakes seen in part (c) were generally due to either incorrect substitution of a point into the equation, or substitution of the vertex coordinates, which got the candidates nowhere.

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

33a. Find the values of $k$ such that $f(x) = 0$ has two equal roots.

**Markscheme**

**METHOD 1**

evidence of discriminant  \( \text{\textit{(MI)}} \)

e.g. $b^2 - 4ac$, discriminant = 0

correct substitution into discriminant  \( A1 \)

e.g. $k^2 - 4 \times \frac{1}{2} \times 8$, $k^2 - 16 = 0$

$k = \pm 4$  \( A1A1 \)  \( N3 \)

**METHOD 2**

recognizing that equal roots means perfect square  \( \text{\textit{(R1)}} \)

e.g. attempt to complete the square, $\frac{1}{2} \left(x^2 + 2kx + 16\right)$

correct working

e.g. $\frac{1}{2}(x + k)^2$, $\frac{1}{2}k^2 = 8$  \( A1 \)

$k = \pm 4$  \( A1A1 \)  \( N3 \)

[4 marks]
Examiners report
A good number of candidates were successful in using the discriminant to find the correct values of \( k \) in part (a), however, there were many who tried to use the quadratic formula without recognizing the significance of the discriminant.

33b. Each value of \( k \) is equally likely for \(-5 \leq k \leq 5\). Find the probability that \( f(x) = 0 \) has no roots. \([4\text{ marks}]\)

Markscheme
- evidence of appropriate approach \((MI)\)
- e.g. \( b^2 - 4ac < 0 \)
- correct working for \( k \) \( A1 \)
- e.g. \(-4 < k < 4 \), \( k^2 < 16 \), list all correct values of \( k \)
- \( p = \frac{7}{11} \) \( A2\) \( N3 \)

Examiners report
Part (b) was very poorly done by nearly all candidates. Common errors included finding the wrong values for \( k \), and not realizing that there were 11 possible values for \( k \).

Let \( f(x) = 2x + 4 \) and \( g(x) = 7x^2 \).

34a. Find \( f^{-1}(x) \). \([3\text{ marks}]\)

Markscheme
- interchanging \( x \) and \( y \) (may be seen at any time) \((MI)\)
- evidence of correct manipulation \((AI)\)
- e.g. \( x = 2y + 4 \)
- \( f^{-1}(x) = \frac{x-4}{2} \) (accept \( y = \frac{x-4}{2}, \frac{x-4}{2} \)) \( A1\) \( N2 \)

Examiners report
All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

34b. Find \( (f \circ g)(x) \). \([2\text{ marks}]\)

Markscheme
- attempt to form composite (in any order) \((MI)\)
- e.g. \( f(7x^2), 2(7x^2) + 4, 7(2x + 4)^2 \)
- \( (f \circ g)(x) = 14x^2 + 4 \) \( A1\) \( N2 \)
Examiners report
All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

34c. Find \((f \circ g)(3.5)\). [2 marks]

**Markscheme**
correct substitution \((AI)\)
e.g. \(7 \times 3.5^2, 14(3.5)^2 + 4\)
\((f \circ g)(3.5) = 175.5\) (accept 176) \(AI \ N2\)

Examiners report
All parts of this question were well answered by most of the candidates. Some misunderstood part (a) and found the derivative or the reciprocal, indicating they were not familiar with the notation for an inverse function. Occasionally, the composition symbol was mistaken for multiplication. Additionally, some candidates composed in the incorrect order.

Jose takes medication. After \(t\) minutes, the concentration of medication left in his bloodstream is given by \(A(t) = 10(0.5)^{0.014t}\), where \(A\) is in milligrams per litre.

35a. Write down \(A(0)\). [1 mark]

**Markscheme**
\(A(0) = 10\) \(AI \ N1\)

[1 mark]

Examiners report
For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

35b. Find the concentration of medication left in his bloodstream after 50 minutes. [2 marks]

**Markscheme**
substitution into formula \((AI)\)
e.g. \(10(0.5)^{0.014(50)}, A(50)\)
\(A(50) = 6.16\) \(AI \ N2\)

[2 marks]
Examiners report
For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

35c. At 13:00, when there is no medication in Jose’s bloodstream, he takes his first dose of medication. He can take his medication [5 marks] again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again?

Markscheme
set up equation \((M1)\)
e.g. \(A(t) = 0.395\)

attempts to solve \((M1)\)
e.g. graph, use of logs

correct working \((A1)\)
e.g. sketch of intersection, \(0.014t \log 0.5 = \log 0.0395\)

\[t = 333.00025 \ldots \quad A1\]
correct time 18:33 or 18:34 (accept 6:33 or 6:34 but nothing else) \(A1 \quad N3\)

[5 marks]

Examiners report
For a later question in Section A, a pleasing number of candidates made good progress. Some candidates believed that raising a base to the zero power gave zero which indicated that they most likely did not begin by analysing the function with their GDC. For part (c), many candidates could set up the equation correctly and had some idea to apply logarithms but became lost in the algebra. Those who used their GDC to find when the function equalled 0.395 typically did so successfully. A common error for those who obtained a correct value for time in minutes was to treat 5.55 hours as 5 hours and 55 minutes after 13:00.

Let \(f(t) = 2t^2 + 7\), where \(t > 0\). The function \(v\) is obtained when the graph of \(f\) is transformed by

a stretch by a scale factor of \(\frac{1}{3}\) parallel to the \(y\)-axis,

followed by a translation by the vector \(\left( \begin{array}{c} 2 \\ -4 \end{array} \right)\).

36a. Find \(v(t)\), giving your answer in the form \(a(t - b)^2 + c\). [4 marks]

Markscheme
applies vertical stretch parallel to the \(y\)-axis factor of \(\frac{1}{3}\) \((M1)\)
e.g. multiply by \(\frac{1}{3}\), \(\frac{1}{3} f(t) = \frac{1}{3} \times 2\)
applies horizontal shift 2 units to the right \((M1)\)
e.g. \(f(t - 2) = t - 2\)
applies a vertical shift 4 units down \((M1)\)
e.g. subtracting 4, \(f(t) - 4 = \frac{2}{3} - 4\)

\[v(t) = \frac{2}{3}(t - 2)^2 - \frac{5}{3} \quad A1 \quad N4\]

[4 marks]
Examiners report
While a number of candidates had an understanding of each transformation, most had difficulty applying them in the correct order, and few obtained the completely correct answer in part (a). Many earned method marks for discerning three distinct transformations. Few candidates knew to integrate to find the distance travelled. Many instead substituted time values into the velocity function or its derivative and subtracted. A number of those who did recognize the need for integration attempted an analytic approach rather than using the GDC, which often proved unsuccessful.

36b. A particle moves along a straight line so that its velocity in ms$^{-1}$, at time $t$ seconds, is given by $v$. Find the distance the particle travels between $t = 5.0$ and $t = 6.8$. [3 marks]

Markscheme
recognizing that distance travelled is area under the curve $M1$

e.g. $\int v \, dt\left\{ \frac{2}{5}(t - 2)^3 - \frac{5}{2}t \right\}$, sketch

distance = 15.576 (accept 15.6) $A2 \quad N2$

Examiners report
While a number of candidates had an understanding of each transformation, most had difficulty applying them in the correct order, and few obtained the completely correct answer in part (a). Many earned method marks for discerning three distinct transformations. Few candidates knew to integrate to find the distance travelled. Many instead substituted time values into the velocity function or its derivative and subtracted. A number of those who did recognize the need for integration attempted an analytic approach rather than using the GDC, which often proved unsuccessful.

37a. Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$. [4 marks]

(i) Find $u_4$.

(ii) Find the sum of the infinite sequence.

Markscheme
(i) correct approach $(A1)$

e.g. $u_4 = (40)\left( \frac{1}{2} \right)^4$, listing terms

$u_4 = 5 \quad A1 \quad N2$

(ii) correct substitution into formula for infinite sum $(A1)$

e.g. $S_\infty = \frac{40}{1-\frac{1}{2}}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80 \quad A1 \quad N2$

[4 marks]
Examiners report
Most candidates found part (a) straightforward, although a common error in (a)(ii) was to calculate 40 divided by $\frac{1}{2}$ as 20.

37b. Consider an arithmetic sequence with $n$ terms, with first term $(-36)$ and eighth term $(-8)$.

(i) Find the common difference.
(ii) Show that $S_n = 2n^2 - 38n$.

Markscheme
(i) attempt to set up expression for $u_n$ \((M1)\)
e.g. $-36 + (8 - 1)d$
correct working \(A1\)
e.g. $-8 = -36 + (8 - 1)d, \frac{-8 - (-36)}{7}$
d = 4 \(A1\) \(N2\)
(ii) correct substitution into formula for sum \((AI)\)
e.g. $S_n = \frac{n}{2} (2(-36) + (n - 1)4)$
correct working \(A1\)
e.g. $S_n = \frac{n}{2} (4n - 76), -36n + 2n^2 - 2n$
$S_n = 2n^2 - 38n$ \(AG\) \(N0\)

[5 marks]

Examiners report
In part (b), some candidates had difficulty with the "show that" and worked backwards from the answer given.

37c. The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find $n$.

Markscheme
multiplying $S_n$ (AP) by 2 or dividing $S$ (infinite GP) by 2 \((M1)\)
e.g. $2S_n, \frac{S_n}{2}, 40$
evidence of substituting into $2S_n = S_\infty$ \(A1\)
e.g. $2n^2 - 38n = 40, 4n^2 - 76n - 80 = 0$\(= 0\)
attempt to solve their quadratic (equation) \((M1)\)
e.g. intersection of graphs, formula
\(n = 20\) \(A2\) \(N3\)

[5 marks]

Examiners report
Most candidates obtained the correct equation in part (c), although some did not reject the negative value of $n$ as impossible in this context.
Let \( f(x) = \frac{20e^{-0.3x}}{e^{0.3x}} \), for \( 0 \leq x \leq 20 \).

38a. Sketch the graph of \( f \).  

[3 marks]

**Markscheme**

Note: Award \( AI \) for approximately correct shape with inflexion/change of curvature, \( AI \) for maximum skewed to the left, \( AI \) for asymptotic behaviour to the right.  

[3 marks]

**Examiners report**
Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

38b. (i) Write down the \( x \)-coordinate of the maximum point on the graph of \( f \).  

(ii) Write down the interval where \( f \) is increasing.  

[3 marks]

**Markscheme**

(i) \( x = 3.33 \) \( AI \) \( NI \)  

(ii) correct interval, with right end point \( 3 \frac{1}{3} \) \( AIAI \) \( N2 \)  

e.g. \( 0 < x \leq 3.33, 0 \leq x < 3 \frac{1}{3} \)  

Note: Accept any inequalities in the right direction.  

[3 marks]

**Examiners report**
Many candidates earned the first four marks of the question in parts (a) and (b) for correctly using their GDC to graph and find the maximum value.

38c. Show that \( f'(x) = \frac{20 - 6x}{e^{3x}} \).  

[5 marks]
**Markscheme**

valid approach  \( (M1) \)

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) \( (AI)(AI) \)

e.g. 20, 0.3e^{0.3x} or \(-0.3e^{-0.3x}\)

correct substitution into product or quotient rule \( A1 \)

e.g. \[ e^{-0.3x} \]

correct working \( A1 \)

e.g. \[ e^{0.3x} \]

\[ f'(x) = \frac{20 - 6x}{e^{0.3x}} \ AG \ N0 \]

[5 marks]

**Examiners report**

Most had a valid approach in part (c) using either the quotient or product rule, but many had difficulty applying the chain rule with a function involving \( e \) and simplifying.

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38d. Find the interval where the rate of change of \( f \) is increasing. \[ 4 \text{ marks} \]

**Markscheme**

consideration of \( f' \) or \( f'' \) \( (M1) \)

valid reasoning \( R1 \)

e.g. sketch of \( f' \), \( f'' \) is positive, \( f'' = 0 \), reference to minimum of \( f' \)

correct value \( 6.6666666 \ldots \left( \frac{2}{3} \right) \) \( (AI) \)

correct interval, with both endpoints \( A1.N3 \)

e.g. \( 6.67 < x \leq 20 \), \( \frac{2}{3} \leq x < 20 \)

[4 marks]

**Examiners report**

Part (d) was difficult for most candidates. Although many associated rate of change with derivative, only the best-prepared students had valid reasoning and could find the correct interval with both endpoints.

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Let \( f(x) = 7 - 2x \) and \( g(x) = x + 3 \).

39a. Find \( (g \circ f)(x) \). \[ 2 \text{ marks} \]

**Markscheme**

attempt to form composite \( (M1) \)

e.g. \( g(7 - 2x) \), \( 7 - 2x + 3 \)

\( (g \circ f)(x) = 10 - 2x \ A1.N2 \)

[2 marks]
Examiners report
A majority of candidates found success in the opening question. Common errors in (a) were to give \( f \circ g \) or to multiply \( f \) by \( g \).

39b. Write down \( g^{-1}(x) \). [1 mark]

Markscheme
\[ g^{-1}(x) = x - 3 \quad A1 \quad N1 \]
[1 mark]

Examiners report
For (b) some gave the inverse as the reciprocal function \( \frac{1}{x+3} \), or wrote \( x = y + 3 \).

39c. Find \( (f \circ g^{-1})(5) \). [2 marks]

Markscheme
METHOD 1
valid approach \( (M1) \)
\[ e.g. \quad g^{-1}(5) = 2, \quad f(5) \]
\[ f(2) = 3 \quad A1 \quad N2 \]
METHOD 2
attempt to form composite of \( f \) and \( g^{-1} \) \( (M1) \)
\[ e.g. \quad (f \circ g^{-1})(x) = 7 - 2(x - 3), \quad 13 - 2x \]
\[ (f \circ g^{-1})(5) = 3 \quad A1 \quad N2 \]
[2 marks]

Examiners report
Most candidates chose to find a composite in (c), sometimes making simple errors when working with brackets and a negative sign. Only a handful used the more efficient \( f(2) = 3 \). Additionally, it was not uncommon for candidates to give a correct substitution but not complete the result. Simple expressions such as \( (7 - 2x) + 3 \) should be finished as \( 10 - 2x \).

Consider \( f(x) = 2kx^2 - 4kx + 1 \), for \( k \neq 0 \). The equation \( f(x) = 0 \) has two equal roots.

40a. Find the value of \( k \). [5 marks]
**Markscheme**

valid approach \((M1)\)

e.g. \(b^2 - 4ac, \Delta = 0, (-4k)^2 - 4(2k)(1)\)
correct equation \(A1\)

e.g. \((-4k)^2 - 4(2k)(1) = 0, 16k^2 = 8k, 2k^2 - k = 0\)
correct manipulation \(A1\)

e.g. \(8k(2k - 1), \frac{8k\sqrt{3}}{32}\)

\(k = \frac{1}{2} A2 \quad N3\)

\([5\text{ marks}]\)

**Examiners report**

Those who knew to set the discriminant to zero had little trouble completing part (a). Some knew that having two equal roots means the factors must be the same, and thus surmised that \((x - 1)(x - 1)\) will achieve \((x - 1)(x - 1)\). This is a valid approach, provided the reasoning is completely communicated. Many candidates set \(f = 0\) and used the quadratic formula, which misses the approach entirely.

40b. The line \(y = p\) intersects the graph of \(f\). Find all possible values of \(p\). \([2\text{ marks}]\)

**Markscheme**

recognizing vertex is on the \(x\)-axis \((M1)\)

e.g. \((1, 0)\), sketch of parabola opening upward from the \(x\)-axis

\(p \geq 0 A1 \quad N1\)

\([2\text{ marks}]\)

**Examiners report**

Part (b) proved challenging for most, and was often left blank. Those who considered a graphical interpretation and sketched the parabola found greater success.

The velocity \(v\text{ ms}^{-1}\) of a particle at time \(t\) seconds, is given by \(v = 2t + \cos 2t\), for \(0 \leq t \leq 2\).

41a. Write down the velocity of the particle when \(t = 0\). \([1\text{ mark}]\)

**Markscheme**

\(v = 1 A1 \quad N1\)

\([1\text{ mark}]\)

**Examiners report**

Many candidates gave a correct initial velocity, although a substantial number of candidates answered that \(0 + \cos 0 = 0\).

41b. When \(t = k\), the acceleration is zero.

(i) Show that \(k = \frac{\pi}{4}\).

(ii) Find the exact velocity when \(t = \frac{\pi}{4}\). \([8\text{ marks}]\)
**Markscheme**

(i) \( \frac{dv}{dt} (2t) = 2 \)  \( A1 \)  \
\[ \frac{d}{dt} (\cos{2t}) = -2\sin{2t} \]  \( A1A1 \)  

Note: Award \( A1 \) for coefficient 2 and \( A1 \) for \(-\sin{2t}\).  
Evidence of considering acceleration = 0  \( (M1) \)  
E.g. \( \frac{dv}{dt} = 0, 2 - 2\sin{2t} = 0 \)  
Correct manipulation  \( A1 \)  
E.g. \( \sin{2k} = 1, \sin{2t} = 1 \)  
\( 2k = \frac{\pi}{2} \) (accept \( 2t = \frac{\pi}{2} \))  \( A1 \)  
\( k = \frac{\pi}{4} \)  \( AG \)  \( N0 \)  
(ii) Attempt to substitute \( t = \frac{\pi}{4} \) into \( v \)  \( (M1) \)  
E.g. \( 2 \left( \frac{\pi}{4} \right) + \cos{\left( \frac{2\pi}{4} \right)} \)  
\( v = \frac{\pi}{2} \)  \( A1 \)  \( N2 \)  

[8 marks]

**Examiners report**

For (b), students commonly applied the chain rule correctly to achieve the derivative, and many recognized that the acceleration must be zero. Occasionally a student would use a double-angle identity on the velocity function before differentiating. This is not incorrect, but it usually caused problems when trying to show \( k = \frac{\pi}{4} \). At times students would reach the equation \( \sin{2k} = 1 \) and then substitute the \( \frac{\pi}{4} \), which does not satisfy the “show that” instruction.

41c. When \( t < \frac{\pi}{4} \), \( \frac{dv}{dt} < 0 \) and when \( t > \frac{\pi}{4} \), \( \frac{dv}{dt} > 0 \).  

[4 marks]

Sketch a graph of \( v \) against \( t \).

**Markscheme**

\[ A1AIA2 \]  \( N4 \)

Notes: Award \( A1 \) for \( y \)-intercept at \((0, 1)\), \( A1 \) for curve having zero gradient at \( t = \frac{\pi}{4} \), \( A2 \) for shape that is concave down to the left of \( \frac{\pi}{4} \) and concave up to the right of \( \frac{\pi}{4} \). If a correct curve is drawn without indicating \( t = \frac{\pi}{4} \), do not award the second \( A1 \) for the zero gradient, but award the final \( A2 \) if appropriate. Sketch need not be drawn to scale. Only essential features need to be clear.  

[4 marks]
Examiners report

The challenge in this question is sketching the graph using the information achieved and provided. This requires students to make graphical interpretations, and as typical in section B, to link the early parts of the question with later parts. Part (a) provides the y-intercept, and part (b) gives a point with a horizontal tangent. Plotting these points first was a helpful strategy. Few understood either the notation or the concept that the function had to be increasing on either side of the $\frac{\pi}{4}$, with most thinking that the point was either a max or min. It was the astute student who recognized that the derivatives being positive on either side of $\frac{\pi}{4}$ creates a point of inflexion. Additionally, important points should be labelled in a sketch. Indicating the $\frac{\pi}{4}$ on the x-axis is a requirement of a clear graph. Although students were not penalized for not labelling the $\frac{\pi}{4}$ on the y-axis, there should be a recognition that the point is higher than the y-intercept.

41d. Let $d$ be the distance travelled by the particle for $0 \leq t \leq 1$.

(i) Write down an expression for $d$.

(ii) Represent $d$ on your sketch.

Markscheme

(i) correct expression $A2$

e.g. $\int_0^1 (2t + \cos 2t)dt$, $\left[ t^2 + \frac{\sin 2t}{2} \right]_0^1 + \frac{\sin^2 1}{2}$, $\int_0^1 vdt$

(ii)

Note: The line at $t = 1$ needs to be clearly after $t = \frac{\pi}{4}$.

[3 marks]

Examiners report

While some candidates recognized that the distance is the area under the velocity graph, surprisingly few included neither the limits of integration in their expression, nor the “$dv$”. Most unnecessarily attempted to integrate the function, often giving an answer with “+C”, and only earned marks if the limits were included with their result. Few recognized that a shaded area is an adequate representation of distance on the sketch, with most fruitlessly attempting to graph a new curve.

Let $f(x) = 3x^2$. The graph of $f$ is translated 1 unit to the right and 2 units down. The graph of $g$ is the image of the graph of $f$ after this translation.

42a. Write down the coordinates of the vertex of the graph of $g$.

Markscheme

(1, −2) $A1A1$ $N2$
42b. Express \( g(x) \) in the form \( g(x) = 3(x - p)^2 + q \).

**Markscheme**

\[ g(x) = 3(x - 1)^2 - 2 \, (\text{accept } p = 1, q = -2) \quad A1A1 \quad N2 \]

[2 marks]

**Examiners report**

Most candidates had little difficulty with this question.

42c. The graph of \( h \) is the reflection of the graph of \( g \) in the \( x \)-axis.

Write down the coordinates of the vertex of the graph of \( h \).

**Markscheme**

\( (1, 2) \quad A1A1 \quad N2 \)

[2 marks]

**Examiners report**

Most candidates had little difficulty with this question. In part (c), a few reflected the vertex in the \( y \)-axis rather than the \( x \)-axis.

43. Let \( f(x) = \cos(x^2) \) and \( g(x) = e^x \), for \(-1.5 \leq x \leq 0.5\).

Find the area of the region enclosed by the graphs of \( f \) and \( g \).

**Markscheme**

evidence of finding intersection points \( \quad (MI) \)
e.g. \( f(x) = g(x), \cos x^2 = e^x \), sketch showing intersection
\( x = -1.11, x = 0 \) (may be seen as limits in the integral) \( A1A1 \)
evidence of approach involving integration and subtraction (in any order) \( \quad (MI) \)
e.g. \( \int_{-1.11}^0 \cos x^2 - e^x \, dx + \int (\cos x^2 - e^x) \, dx = g - f \)

area = 0.282 \quad A2 \quad N3

[6 marks]

**Examiners report**

This question was poorly done by a great many candidates. Most seemed not to understand what was meant by the phrase "region enclosed by" as several candidates assumed that the limits of the integral were those given in the domain. Few realized what area was required, or that intersection points were needed. Candidates who used their GDCs to first draw a suitable sketch could normally recognize the required region and could find the intersection points correctly. However, it was disappointing to see the number of candidates who could not then use their GDC to find the required area or who attempted unsuccessful analytical approaches.
The following diagram shows the graph of \( f(x) = e^{-x^2} \).

The points A, B, C, D and E lie on the graph of \( f \). Two of these are points of inflexion.

44a. Identify the **two** points of inflexion.  

**Markscheme**  
B, D  **A1 A1**  **N2**  
**[2 marks]**

**Examiners report**  
Most candidates were able to recognize the points of inflexion in part (a).

44b. (i) Find \( f'(x) \).  

(ii) Show that \( f''(x) = (4x^2 - 2)e^{-x^2} \).

**Markscheme**  
(i) \( f'(x) = -2xe^{-x^2} \)  **A1A1**  **N2**  
**Note:** Award **A1** for \( e^{-x^2} \) and **A1** for \(-2x\).  
(ii) finding the derivative of \(-2x\), i.e. \(-2\)  **(A1)**  
evidence of choosing the product rule  **(MI)**  
e.g. \( -2e^{-x^2} -2x \times 2xe^{-x^2} \)  
\(-2e^{-x^2} + 4x^2e^{-x^2} \)  **A1**  
\( f''(x) = (4x^2 - 2)e^{-x^2} \)  **AG**  **N0**  
**[5 marks]**

**Examiners report**  
Most candidates were able to recognize the points of inflexion in part (a) and had little difficulty with the first and second derivatives in part (b). A few did not recognize the application of the product rule in part (b).

44c. Find the \( x \)-coordinate of each point of inflexion.  

**[4 marks]**
Markscheme
valid reasoning  \( R1 \)
e.g. \( f''(x) = 0 \)

attempts to solve the equation  \( (M1) \)
e.g. \( (4x^2 - 2) = 0 \), sketch of \( f''(x) \)

\[
p = 0.707 \left( \frac{1}{\sqrt{2}} \right), \quad q = -0.707 \left( -\frac{1}{\sqrt{2}} \right)
\]

\( A1A1 \)  \( N3 \)

[4 marks]

Examiners report
Obtaining the \( x \)-coordinates of the inflexion points in (c) usually did not cause many problems.

44d. Use the second derivative to show that one of these points is a point of inflexion.  \([4\,\text{marks}]\)

Markscheme
evidence of using second derivative to test values on either side of POI  \( M1 \)
e.g. finding values, reference to graph of \( f'' \), sign table
correct working  \( A1A1 \)
e.g. finding any two correct values either side of POI,
checking sign of \( f'' \) on either side of POI
reference to sign change of \( f''(x) \)  \( R1 \)  \( N0 \)

[4 marks]

Examiners report
Only the better-prepared candidates understood how to set up a second derivative test in part (d). Many of those did not show, or clearly indicate, the values of \( x \) used to test for a point of inflexion, but merely gave an indication of the sign. Some candidates simply resorted to showing that \( f'' \left( \pm \frac{1}{\sqrt{2}} \right) = 0 \), completely missing the point of the question. The necessary condition for a point of inflexion, i.e. \( f''(x) = 0 \) and the change of sign for \( f''(x) \), seemed not to be known by the vast majority of candidates.

Let \( f(x) = \log_3 \frac{5}{2} + \log_3 16 - \log_3 4 \), for \( x > 0 \).

45a. Show that \( f(x) = \log_3 2x \).  \([2\,\text{marks}]\)

Markscheme
combining 2 terms  \( (A1) \)
e.g. \( \log_3 8x - \log_3 4 \), \( \log_3 \frac{1}{2} x + \log_3 4 \)
expression which clearly leads to answer given  \( A1 \)
e.g. \( \log_3 \frac{8x}{4}, \log_3 \frac{4x}{2} \)

\( f(x) = \log_3 2x \)  \( AG \)  \( N0 \)

[2 marks]
Examiners report
Few candidates had difficulty with part (a) although it was often communicated using some very sloppy applications of the rules of logarithm, writing \( \log_{16} \frac{16}{4} \) instead of \( \log \left( \frac{16}{4} \right) \).

45b. Find the value of \( f(0.5) \) and of \( f(4.5) \). [3 marks]

Markscheme
attempt to substitute either value into \( f \) (MI)
e.g. \( \log_{1} 1 \), \( \log_{9} 9 \)
\( f(0.5) = 0 \), \( f(4.5) = 2 \) AI A1 N3

Examiners report
Part (b) was generally done well.

45c. The function \( f \) can also be written in the form \( f(x) = \frac{\ln a x}{\ln b} \). [6 marks]

(i) Write down the value of \( a \) and of \( b \).
(ii) Hence on graph paper, sketch the graph of \( f \), for \(-5 \leq x \leq 5 \), \(-5 \leq y \leq 5 \), using a scale of 1 cm to 1 unit on each axis.
(iii) Write down the equation of the asymptote.

Markscheme
(i) \( a = 2 \), \( b = 3 \) A1 AI N1 N1
(ii)

Note: Award A1 for sketch approximately through \((0.5 \pm 0.1, 0 \pm 0.1)\), AI for approximately correct shape, AI for sketch asymptotic to the y-axis.

(iii) \( x = 0 \) (must be an equation) AI N1

[6 marks]
Examiners report
Part (c) (i) was generally done well; candidates seemed quite comfortable changing bases. There were some very good sketches in (c) (ii), but there were also some very poor ones with candidates only considering shape and not the location of the $x$-intercept or the asymptote. A surprising number of candidates did not use the scale required by the question and/or did not use graph paper to sketch the graph. In some cases, it was evident that students simply transposed their graphs from their GDC without any analytical consideration.

45d. Write down the value of $f^{-1}(0)$.

Markscheme
\[ f^{-1}(0) = 0.5 \quad A1 \quad N1 \]

[1 mark]

Examiners report
Part (d) was poorly done as candidates did not consider the command term, “write down” and often proceeded to find the inverse function before making the appropriate substitution.

45e. The point A lies on the graph of $f$. At A, $x = 4.5$.

On your diagram, sketch the graph of $f^{-1}$, noting clearly the image of point A.

Markscheme

Note: Award $A1$ for sketch approximately through $(0 \pm 0.1, 0.5 \pm 0.1)$, $A1$ for approximately correct shape of the graph reflected over $y = x$, $A1$ for sketch asymptotic to x-axis, $A1$ for point $(2 \pm 0.1, 4.5 \pm 0.1)$ clearly marked and on curve.

[4 marks]
**Examiners report**

Part (e) eluded a great many candidates as most preferred to attempt to find the inverse analytically rather than simply reflecting the graph of \( f \) in the line \( y = x \). This graph also suffered from the same sort of problems as the graph in (c) (ii). Some students did not have their curve passing through \((2, 4.5)\) nor did they clearly indicate its position as instructed. This point was often mislabelled on the graph of \( f \). The efforts in this question demonstrated that students often work tenuously from one question to the next, without considering the "big picture", thereby failing to make important links with earlier parts of the question.

Let \( f(x) = 3\ln x \) and \( g(x) = \ln 5x^3 \).

46a. Express \( g(x) \) in the form \( f(x) + \ln a \), where \( a \in \mathbb{Z}^+ \). [4 marks]

**Markscheme**

- attempt to apply rules of logarithms \((MI)\)
- e.g. \( \ln a^b = b \ln a \), \( \ln ab = \ln a + \ln b \)
- correct application of \( \ln a^b = b \ln a \) (seen anywhere) \( A1 \)
- e.g. \( 3\ln x = \ln x^3 \)
- correct application of \( \ln ab = \ln a + \ln b \) (seen anywhere) \( A1 \)
- e.g. \( \ln 5x^3 = \ln 5 + \ln x^3 \)
- so \( \ln 5x^3 = \ln 5 + 3\ln x \)
- \( g(x) = f(x) + \ln 5 \) (accept \( g(x) = 3\ln x + \ln 5 \)) \( A1 \) \( N1 \)

[4 marks]

**Examiners report**

This question was very poorly done by the majority of candidates. While candidates seemed to have a vague idea of how to apply the rules of logarithms in part (a), very few did so successfully. The most common error in part (a) was to begin incorrectly with \( \ln 5x^3 = 3\ln 5x \). This error was often followed by other errors.

46b. The graph of \( g \) is a transformation of the graph of \( f \). Give a full geometric description of this transformation. [3 marks]

**Markscheme**

- transformation with correct name, direction, and value \( A3 \)
- e.g. translation by \( \begin{pmatrix} 0 \\ \ln 5 \end{pmatrix} \), shift up by \( \ln 5 \), vertical translation of \( \ln 5 \)

[3 marks]

**Examiners report**

In part (b), very few candidates were able to describe the transformation as a vertical translation (or shift). Many candidates attempted to describe numerous incorrect transformations, and some left part (b) entirely blank.
The following diagram shows part of the graph of a quadratic function \( f \).

\[ f(x) = -10(x - p)(x - q) \]

Markscheme

\[ f(x) = -10(x + 4)(x - 6) \quad A1 \quad A1 \quad N2 \]

[2 marks]

Examiners report

Parts (a) and (c) of this question were very well done by most candidates.

\[ f(x) = -10(x - h)^2 + k \]

Markscheme

METHOD 1

attempting to find the \( x \)-coordinate of maximum point \((M1)\)

e.g. averaging the \( x \)-intercepts, sketch, \( y' = 0 \), axis of symmetry

attempting to find the \( y \)-coordinate of maximum point \((M1)\)

e.g. \( k = -10(1 + 4)(1 - 6) \)

\[ f(x) = -10(x - 1)^2 + 250 \quad A1 \quad A1 \quad N4 \]

METHOD 2

attempt to expand \( f(x) \) \((M1)\)

e.g. \(-10(x^2 - 2x - 24)\)

attempt to complete the square \((M1)\)

e.g. \(-10((x - 1)^2 - 1 - 24)\)

\[ f(x) = -10(x - 1)^2 + 250 \quad A1 \quad A1 \quad N4 \]

[4 marks]
47c. Show that \( f(x) \) can also be written in the form \( f(x) = 240 + 20x - 10x^2 \). [2 marks]

**Markscheme**
- attempt to simplify \((M1)\)
  - e.g. distributive property, \(-10(x - 1)(x - 1) + 250\)
  - correct simplification \(A1\)
  - e.g. \(-10(x^2 - 6x + 4x - 24) , -10(x^2 - 2x + 1) + 250\)
- \(f(x) = 240 + 20x - 10x^2\) \(AG\) \(N0\) [2 marks]

**Examiners report**
Parts (a) and (c) of this question were very well done by most candidates.

47d. A particle moves along a straight line so that its velocity, \( v \text{ ms}^{-1} \), at time \( t \) seconds is given by \( v = 240 + 20t - 10t^2 \), for \( 0 \leq t \leq 6 \).

(i) Find the value of \( t \) when the speed of the particle is greatest.

(ii) Find the acceleration of the particle when its speed is zero.

**Markscheme**
- (i) valid approach \((M1)\)
  - e.g. vertex of parabola, \( v'(t) = 0\)
  - \( t = 1 \) \(A1\) \(N2\)
- (ii) recognizing \( a(t) = v'(t) \) \((M1)\)
  - \( a(t) = 20 - 20t \) \(A1A1\)
  - speed is zero \( \Rightarrow t = 6 \) \((A1)\)
  - \( a(6) = -100 \text{ (ms}^{-2}\) \(A1\) \(N3\) [7 marks]

**Examiners report**
In part (d), it was clear that many candidates were not familiar with the relationship between velocity and acceleration, and did not understand how those concepts were related to the graph which was given. A large number of candidates used time \( t = 1 \) in part b(ii), rather than \( t = 6 \). To find the acceleration, some candidates tried to integrate the velocity function, rather than taking the derivative of velocity. Still others found the derivative in part b(i), but did not realize they needed to use it in part b(ii), as well.

Let \( f(x) = 3x \), \( g(x) = 2x - 5 \) and \( h(x) = (f \circ g)(x) \).

48a. Find \( h(x) \). [2 marks]
48b. Find \( h^{-1}(x) \). 

**Markscheme**

interchanging \( x \) and \( y \)  \( (M1) \)

evidence of correct manipulation  \( (AI) \)

e.g. \( y + 15 = 6x \implies y = \frac{x - 15}{6} \)

\[ h^{-1}(x) = \frac{x + 15}{6} \quad A1 \quad N3 \]

[3 marks]

**Examiners report**

Most candidates handled this question with ease. Some were not familiar with the notation of composite functions assuming that \( (f \circ g)(x) \) implied finding the composition and then multiplying this by \( x \). Others misunderstood part (b) and found the reciprocal function or the derivative, indicating they were not familiar with the notation for an inverse function.

Let \( f(x) = x^2 + 4 \) and \( g(x) = x - 1 \).

49a. Find \( (f \circ g)(x) \).  

**Markscheme**

attempt to form composition (in any order)  \( (M1) \)

\( (f \circ g)(x) = (x - 1)^2 + 4 \quad (x^2 - 2x + 5) \quad A1 \quad N2 \)

[2 marks]

**Examiners report**

Candidates showed good understanding of finding the composite function in part (a).

49b. The vector \( \begin{pmatrix} 3 \\ -1 \end{pmatrix} \) translates the graph of \( (f \circ g) \) to the graph of \( h \).

Find the coordinates of the vertex of the graph of \( h \).
**Markscheme**

**METHOD 1**
vertex of \(f \circ g\) at \((1, 4)\) \(\text{(AI)}\)
evidence of appropriate approach \(\text{(MI)}\)
e.g. adding \(\begin{pmatrix} 3 \\ -1 \end{pmatrix}\) to the coordinates of the vertex of \(f \circ g\)
vertex of \(h\) at \((4, 3)\) \(\text{AI} \quad \text{N3}\)

**METHOD 2**
attempt to find \(h(x)\) \(\text{(MI)}\)
e.g. \([(x - 3) - 1]^2 + 4 - 1\), \(h(x) = (f \circ g)(x - 3) - 1\)
\(h(x) = (x - 4)^2 + 3\) \(\text{(AI)}\)
vertex of \(h\) at \((4, 3)\) \(\text{AI} \quad \text{N3}\)

[3 marks]

**Examiners report**
There were some who did not seem to understand what the vector translation meant in part (b).

49c. The vector \(\begin{pmatrix} 3 \\ -1 \end{pmatrix}\) translates the graph of \((f \circ g)\) to the graph of \(h\).

Show that \(h(x) = x^2 - 8x + 19\).

**Markscheme**
evidence of appropriate approach \(\text{(MI)}\)
e.g. \((x - 4)^2 + 3, (x - 3)^2 - 2(x - 3) + 5 - 1\)
simplifying \(\text{AI}\)
e.g. \(h(x) = x^2 - 8x + 16 + 3, x^2 - 6x + 9 - 2x + 6 + 4\)
\(h(x) = x^2 - 8x + 19\) \(\text{AG} \quad \text{N0}\)

[2 marks]

**Examiners report**
Candidates showed good understanding of manipulating the quadratic in part (c).

49d. The vector \(\begin{pmatrix} 3 \\ -1 \end{pmatrix}\) translates the graph of \((f \circ g)\) to the graph of \(h\).

The line \(y = 2x - 6\) is a tangent to the graph of \(h\) at the point \(P\). Find the \(x\)-coordinate of \(P\).
Markscheme

METHOD 1
equating functions to find intersection point  \((M1)\)
e.g. \(x^2 - 8x + 19 = 2x - 6, y = h(x)\)
\(x^2 - 10x + 25 + 0\)  \(A1\)
evidence of appropriate approach to solve  \((M1)\)
e.g. factorizing, quadratic formula
appropriate working  \(A1\)
e.g. \((x - 5)^2 = 0\)
ex = 5  \(p = 5\)  \(A1\)  \(N3\)

METHOD 2
attempt to find \(h'(x)\)  \((M1)\)
h(x) = 2x - 8  \(A1\)
recognizing that the gradient of the tangent is the derivative  \((M1)\)
e.g. gradient at \(p = 2\)
2x - 8 = 2 (2x = 10)  \(A1\)
x = 5  \(A1\)  \(N3\)

[5 marks]

Examiners report
There was more than one method to solve for \(h\) in part (d), and a pleasing number of candidates were successful in this part of the question.

Let \(f(x) = 8x - 2x^2\). Part of the graph of \(f\) is shown below.

Let \(f(x) = 8x - 2x^2\). Part of the graph of \(f\) is shown below.

50a. Find the \(x\)-intercepts of the graph.  \([4\ \text{marks}]\)
**Markscheme**

- Evidence of setting function to zero \((M1)\)
  - e.g. \(f(x) = 0, \ 8x = 2x^2\)
- Evidence of correct working \(A1\)
  - e.g. \(0 = 2x(4 - x) \cdot \frac{6 + \sqrt{5}}{4}\)
  - \(x\)-intercepts are at 4 and 0 (accept \((4, 0)\) and \((0, 0)\), or \(x = 4, x = 0\) \(A1A1 \ N1N1\)

\([4 \text{ marks}]\)

**Examiners report**

This question was answered well by most candidates.

---

50b. (i) Write down the equation of the axis of symmetry. \([3 \text{ marks}]\)

(ii) Find the \(y\)-coordinate of the vertex.

**Markscheme**

(i) \(x = 2\) (must be equation) \(AI \ N1\)
(ii) Substituting \(x = 2\) into \(f(x)\) \((M1)\)

\(y = 8 \ A1 \ N2\)

\([3 \text{ marks}]\)

**Examiners report**

This question was answered well by most candidates. Some did not give an equation for their axis of symmetry.

---

Let \(f(x) = \log_3 \sqrt{x}\) for \(x > 0\).

51a. Show that \(f^{-1}(x) = 3^{2x}\). \([2 \text{ marks}]\)

**Markscheme**

Interchanging \(x\) and \(y\) (seen anywhere) \((M1)\)
- e.g. \(x = \log \sqrt{y}\) (accept any base)
- Evidence of correct manipulation \(AI\)
  - e.g. \(3^x = \sqrt{y}, \ 3^x = x^{\frac{1}{2}}, \ x = \frac{1}{2} \log_3 y, \ 2y = \log_3 x\)
  - \(f^{-1}(x) = 3^{2x} \ AG \ N0\)

\([2 \text{ marks}]\)

**Examiners report**

Candidates were generally skilled at finding the inverse of a logarithmic function.

---

51b. Write down the range of \(f^{-1}\). \([1 \text{ mark}]\)
Markscheme

\[ y > 0, \quad f^{-1}(x) > 0 \quad A1 \quad N1 \]

[1 mark]

Examiners report

Few correctly gave the range of this function, often stating “all real numbers” or “\( y \geq 0 \)”, missing the idea that the range of an inverse is the domain of the original function.

---

51c. Let \( g(x) = \log_3 x \), for \( x > 0 \).

Find the value of \( (f^{-1} \circ g)(2) \), giving your answer as an integer.

Markscheme

METHOD 1

finding \( g(2) = \log_3 2 \) (seen anywhere) \( A1 \)

attempt to substitute \( (MI) \)

e.g. \( (f^{-1} \circ g)(2) = 3^{2\log_3 2} \)

evidence of using log or index rule \( (A1) \)

e.g. \( (f^{-1} \circ g)(2) = 3^{\log_3 2} \cdot 3^{\log_3 2} \)

\( (f^{-1} \circ g)(2) = 4 \quad A1 \quad N1 \)

METHOD 2

attempt to form composite (in any order) \( (MI) \)

e.g. \( (f^{-1} \circ g)(x) = 3^{2\log_3 x} \)

evidence of using log or index rule \( (A1) \)

e.g. \( (f^{-1} \circ g)(x) = 3^{\log_3 x^2} \cdot 3^{\log_3 x^2} \)

\( (f^{-1} \circ g)(x) = x^2 \quad A1 \)

\( (f^{-1} \circ g)(2) = 4 \quad A1 \quad N1 \)

[4 marks]

Examiners report

Some candidates answered part (c) correctly, although many did not get beyond \( 3^{2\log_3 2} \). Some attempted to form the composite in the incorrect order. Others interpreted \( (f^{-1} \circ g)(2) \) as multiplication by 2.
Let \( f(x) = \frac{1}{2}x^3 - x^2 - 3x \). Part of the graph of \( f \) is shown below.

There is a maximum point at \( A \) and a minimum point at \( B(3, -9) \).

52a. Find the coordinates of \( A \). [8 marks]

**Markscheme**

\[ f(x) = x^2 - 2x - 3 \]  
\( A1A1A1 \)  
evidence of solving \( f'(x) = 0 \) \( (M1) \)  
e.g. \( x^2 - 2x - 3 = 0 \)  
evidence of correct working \( A1 \)  
e.g. \((x + 1)(x - 3)\), \( \frac{2 \pm \sqrt{16}}{2} \)  
\( x = -1 \) (ignore \( x = 3 \)) \( (A1) \)  
evidence of substituting their negative \( x \)-value into \( f(x) \) \( (M1) \)  
e.g. \( \frac{1}{2}(-1)^3 - (-1)^2 - 3(-1), -\frac{1}{3} - 1 + 3 \)  
\( y = \frac{5}{3} \) \( A1 \)  
coordinates are \( \left(-1, \frac{5}{3}\right) \) \( N3 \)  
[8 marks]

**Examiners report**

A majority of candidates answered part (a) completely.

52b. Write down the coordinates of [6 marks]

(i) the image of \( B \) after reflection in the \( y \)-axis;

(ii) the image of \( B \) after translation by the vector \( \begin{pmatrix} -2 \\ 5 \end{pmatrix} \);

(iii) the image of \( B \) after reflection in the \( x \)-axis followed by a horizontal stretch with scale factor \( \frac{1}{4} \).
**Markscheme**

(i) \((-3, -9)\) \(A1\) \(N1\)

(ii) \((1, -4)\) \(A1A1\) \(N2\)

(iii) reflection gives \((3, 9)\) \((AI)\)

stretch gives \(\left(\frac{3}{2}, 9\right)\) \(A1A1\) \(N3\)

\([6 \text{ marks}]\)

**Examiners report**

Candidates were generally successful in finding images after single transformations in part (b). Common incorrect answers for (biii) included \((-3, -9), (1, -4)\) and \((6, 9)\), demonstrating difficulty with images from horizontal stretches.

Let \(f(x) = p(x - q)(x - r)\). Part of the graph of \(f\) is shown below.

![Graph of f(x)](image)

The graph passes through the points \((-2, 0), (0, -4)\) and \((4, 0)\).

53a. Write down the value of \(q\) and of \(r\). \([2 \text{ marks}]\)

**Markscheme**

\(q = -2, r = 4\) or \(q = 4, r = -2\) \(A1A1\) \(N2\)

\([2 \text{ marks}]\)

**Examiners report**

The majority of candidates were successful on some or all parts of this question, with some candidates using a mix of algebra and graphical reasoning and others ignoring the graph and working only algebraically. Some did not recognize that \(p\) and \(q\) are the roots of the quadratic function and hence gave the answers as 2 and \(-4\).

53b. Write down the equation of the axis of symmetry. \([1 \text{ mark}]\)

**Markscheme**

\(x = 1\) (must be an equation) \(A1\) \(N1\)

\([1 \text{ mark}]\)
Examiners report

A common error in part (b) was the absence of an equation. Some candidates wrote down the equation \( x = \frac{1}{2} \) but were not able to substitute correctly. Those students did not realize that the axis of symmetry is always halfway between the \( x \)-intercepts.

53c. Find the value of \( p \). [3 marks]

Markscheme

substituting \((0, -4)\) into the equation \((M1)\)
e.g. \(-4 = p(0 - (-2))(0 - 4)\), \(-4 = p(-4)(2)\)
correct working towards solution \((AI)\)
e.g. \(-4 = -8p\)
\(p = \frac{4}{8} (= \frac{1}{2}) \) \(AI N2\)

Examiners report

More candidates had trouble with part (c) with erroneous substitutions and simplification mistakes commonplace.

Let \( f(x) = \cos 2x \) and \( g(x) = 2x^2 - 1 \).

54a. Find \( f\left(\frac{x}{2}\right) \). [2 marks]

Markscheme

\( f\left(\frac{x}{2}\right) = \cos \pi \) \( (AI)\)
\( = -1 \) \( AI N2\)

Examiners report

In part (a), a number of candidates were not able to evaluate \( \cos \pi \), either leaving it or evaluating it incorrectly.

54b. Find \( (g \circ f)\left(\frac{x}{2}\right) \). [2 marks]

Markscheme

\( (g \circ f)\left(\frac{x}{2}\right) = g(-1) (= 2(-1)^2 - 1) \) \( (AI)\)
\( = 1 \) \( AI N2\)

Examiners report

Almost all candidates evaluated the composite function in part (b) in the given order, many earning follow-through marks for incorrect answers from part (a). On both parts (a) and (b), there were candidates who correctly used double-angle formulas to come up with correct answers; while this is a valid method, it required unnecessary additional work.
54c. Given that \((g \circ f)(x)\) can be written as \(\cos(kx)\), find the value of \(k, k \in \mathbb{Z}\).  

**Markscheme**

\[(g \circ f)(x) = 2(\cos(2x))^2 - 1 = 2\cos^2(2x) - 1\] \(A1\)  

evidence of \(2\cos^2\theta - 1 = \cos 2\theta\) (seen anywhere) \(M1\)  

\[(g \circ f)(x) = \cos 4x\]  

\[k = 4\] \(A1\) \(N2\)  

[3 marks]

**Examiners report**

Candidates were not as successful in part (c). Many tried to use double-angle formulas, but either used the formula incorrectly or used it to write the expression in terms of \(\cos x\) and went no further. There were a number of cases in which the candidates "accidentally" came up with the correct answer based on errors or lucky guesses and did not earn credit for their final answer. Only a few candidates recognized the correct method of solution.

The velocity \(v\) ms\(^{-1}\) of an object after \(t\) seconds is given by \(v(t) = 15\sqrt{t} - 3t\), for \(0 \leq t \leq 25\).

55a. On the grid below, sketch the graph of \(v\), clearly indicating the maximum point.  

[3 marks]
**Markscheme**

Note: Award A1 for approximately correct shape, A1 for right endpoint at (25, 0) and A1 for maximum point in circle. [3 marks]

**Examiners report**

The graph in part (a) was well done. It was pleasing to see many candidates considering the domain as they sketched their graph.

55b. (i) Write down an expression for \(d\). [4 marks]

(ii) Hence, write down the value of \(d\).

**Markscheme**

(i) recognizing that \(d\) is the area under the curve \(M1\)

e.g. \(\int v(t)\) 

correct expression in terms of \(t\), with correct limits \(A2\) \(N3\)

e.g. \(d = \int_0^2 (15\sqrt{7} - 3t)\,dt\), \(d = \int_0^3 v\,dt\)

(ii) \(d = 148.5\) (m) (accept 149 to 3 sf) \(A1\) \(N1\) [4 marks]

**Examiners report**

Part (b) (i) asked for an expression which bewildered a great many candidates. However, few had difficulty obtaining the correct answer in (b) (ii).

Let \(f'(x) = -24x^3 + 9x^2 + 3x + 1\).

56a. There are two points of inflexion on the graph of \(f\). Write down the \(x\)-coordinates of these points. [3 marks]
**Markscheme**

valid approach  \( R1 \)

e.g. \( f''(x) = 0 \), the max and min of \( f' \) gives the points of inflexion on \( f \)

\(-0.114, 0.364 \) (accept \((-0.114, 0.811)\) and \((0.364, 2.13)\)) \( A1 \)    \( N1 \)

[3 marks]

**Examiners report**

There were mixed results in part (a). Students were required to understand the relationships between a function and its derivative and often obtained the correct solutions with incorrect or missing reasoning.

56b. Let \( g(x) = f''(x) \). Explain why the graph of \( g \) has no points of inflexion. \( [2 \text{ marks}] \)

**Markscheme**

**METHOD 1**

graph of \( g \) is a quadratic function \( R1 \) \( N1 \)

a quadratic function does not have any points of inflexion \( R1 \) \( N1 \)

**METHOD 2**

graph of \( g \) is concave down over entire domain \( R1 \) \( N1 \)

therefore no change in concavity \( R1 \) \( N1 \)

**METHOD 3**

\( g''(x) = -144 \) \( R1 \) \( N1 \)

therefore no points of inflexion as \( g''(x) \neq 0 \) \( R1 \) \( N1 \)

[2 marks]

**Examiners report**

In part (b), the question was worth two marks and candidates were required to make two valid points in their explanation. There were many approaches to take here and candidates often confused their reasoning or just kept writing hoping that somewhere along the way they would say something correct to pick up the points. Many confused \( f' \) and \( f'' \).
Let \( f(x) = x \ln(4 - x^2) \), for \(-2 < x < 2\). The graph of \( f \) is shown below.

The graph of \( f \) crosses the \( x \)-axis at \( x = a \), \( x = 0 \) and \( x = b \).

57a. Find the value of \( a \) and \( b \). [3 marks]

**Markscheme**

evidence of valid approach \( (MI) \)

\( f(x) = 0 \), graph

\( a = -1.73 \), \( b = 1.73 \) \((a = -\sqrt{3}, b = \sqrt{3})\) \( A1A1\) \( N3 \)

[3 marks]

**Examiners report**

This question was well done by many candidates. If there were problems, it was often with incorrect or inappropriate GDC use. For example, some candidates used the trace feature to answer parts (a) and (b), which at best, only provides an approximation.

57b. The graph of \( f \) has a maximum value when \( x = c \). [2 marks]

Find the value of \( c \).

**Markscheme**

attempt to find max \( (MI) \)

\( f'(x) = 0 \), graph

\( c = 1.15 \) \((\text{accept } (1.15, 1.13))\) \( A1\) \( N2 \)

[2 marks]

**Examiners report**

This question was well done by many candidates. If there were problems, it was often with incorrect or inappropriate GDC use. For example, some candidates used the trace feature to answer parts (a) and (b), which at best, only provides an approximation.

57c. The region under the graph of \( f \) from \( x = 0 \) to \( x = c \) is rotated 360° about the \( x \)-axis. Find the volume of the solid formed. [3 marks]
**Markscheme**

attempt to substitute either limits or the function into formula  \( M1 \)

e.g. \( V = \pi \int_0^c |f(x)|^2 \, dx \), \( \pi \int [x \ln(4 - x^2)]^2 \, dx \)

\[ V = 2.16 \quad A2 \quad N2 \]

[3 marks]

**Examiners report**

Most candidates were able to set up correct expressions for parts (c) and (d) and if they had used their calculators, could find the correct answers. Some candidates omitted the important parts of the volume formula. Analytical approaches to (c) and (d) were always futile and no marks were gained.

---

57d. Let \( R \) be the region enclosed by the curve, the \( x \)-axis and the line \( x = c \), between \( x = a \) and \( x = c \).

Find the area of \( R \).  

**Markscheme**

valid approach recognizing 2 regions  \( (M1) \)

e.g. finding 2 areas

correct working  \( (A1) \)

e.g. \( \int_0^{1.73} f(x) \, dx + \int_0^{1.149} f(x) \, dx \), \( -\int_0^{a} f(x) \, dx + \int_0^{b} f(x) \, dx \)

area = 2.07 (accept 2.06)  \( A2 \quad N3 \)

[4 marks]

**Examiners report**

Most candidates were able to set up correct expressions for parts (c) and (d) and if they had used their calculators, could find the correct answers. Some candidates omitted the important parts of the volume formula. Analytical approaches to (c) and (d) were always futile and no marks were gained.

---

Let \( g(x) = \frac{1}{2} x \sin x \), for \( 0 \leq x \leq 4 \).

58a. Sketch the graph of \( g \) on the following set of axes.  

[4 marks]
58b. Hence find the value of $x$ for which $g(x) = -1$.

**Markscheme**

attempting to solve $g(x) = -1$ \((M1)\)
e.g. marking coordinate on graph, $\frac{1}{2}x \sin x + 1 = 0$

$x = 3.71 \ N1$

**Examiners report**

This question was well done by the majority of candidates. Most sketched an approximately correct shape in the given domain, though some candidates did not realize they had to set their GDC to radians, producing a meaningless sketch. Candidates need to be aware that unless otherwise specified, questions will expect radians to be used. The most confident candidates used a table to aid their graphing. Although most recognized the need of the GDC to answer part (b), some used the trace function, hence obtaining an inaccurate result, while others attempted a fruitless analytical approach. Merely stating “using GDC” is insufficient evidence of method; a sketch or an equation set equal to zero are both examples of appropriate evidence.

59a. Write down the number of terms in this expansion.

**Markscheme**

12 terms \(A1\) \(N1\)

**Examiners report**

Consider the expansion of \((x + 2)^{11}\).
Examiners report
Most candidates attempted this question, and many made good progress. A number of candidates spent time writing out Pascal’s triangle in full. Common errors included 11 for part (a) and not writing out the simplified form of the term for part (b). Another common error was adding instead of multiplying the parts of the term in part (b).

59b. Find the term containing $x^2$.

[4 marks]

Markscheme

evidence of binomial expansion \((MI)\)
e.g. \(\binom{n}{r} a^{n-r}b^r\), an attempt to expand, Pascal’s triangle


evidence of choosing correct term \((AI)\)
e.g. 10th term, \(\binom{11}{9}\), \((x)^2(2)^9\)

correct working \(AI\)
e.g. \(\binom{11}{9}\) \((x)^2(2)^9\), \(55 \times 2^9\)

28160$x^2$ \(AI\) \(N2\)

[4 marks]

Examiners report
Most candidates attempted this question, and many made good progress. A number of candidates spent time writing out Pascal’s triangle in full. Common errors included 11 for part (a) and not writing out the simplified form of the term for part (b). Another common error was adding instead of multiplying the parts of the term in part (b).

60. Solve \(\log_2 x + \log_2 (x - 2) = 3\), for \(x > 2\).

[7 marks]

Markscheme

recognizing \(\log a + \log b = \log ab\) (seen anywhere) \((AI)\)
e.g. \(\log_2(x(x - 2))\), \(x^2 - 2x\)

recognizing \(\log_b b = x \Leftrightarrow a^x = b\) \((AI)\)
e.g. \(2^3 = 8\)

correct simplification \(AI\)
e.g. \(x(x - 2) = 2^3, x^2 - 2x - 8\)

evidence of correct approach to solve \((MI)\)
e.g. factorizing, quadratic formula

correct working \(AI\)
e.g. \((x - 4)(x + 2), \frac{2\sqrt{3}}{2}\)

\[x = 4\] \(A2\) \(N3\)

[7 marks]
Examiners report

Candidates secure in their understanding of logarithm properties usually had success with this problem, solving the resulting quadratic either by factoring or using the quadratic formula. The majority of successful candidates correctly rejected the solution that was not in the domain. A number of candidates, however, were unclear on logarithm properties. Some unsuccessful candidates were able to demonstrate understanding of one property but without both were not able to make much progress. A few candidates employed a “guess and check” strategy, but this did not earn full marks.

Let \( f(x) = 6 + 6 \sin x \). Part of the graph of \( f \) is shown below.

![Graph of f(x) = 6 + 6 \sin x]

The shaded region is enclosed by the curve of \( f \), the \( x \)-axis, and the \( y \)-axis.

61a. Solve for \( 0 \leq x < 2\pi \) \[5 \text{ marks}\]

(i) \( 6 + 6 \sin x = 6 \); \( 6 + 6 \sin x = 0 \).

**Markscheme**

(i) \( \sin x = 0 \) \( \text{A1} \)  
\( x = 0 , x = \pi \) \( \text{A1A1 N2} \)

(ii) \( \sin x = -1 \) \( \text{A1} \)  
\( x = \frac{3\pi}{2} \) \( \text{A1 N1} \)

61b. Write down the exact value of the \( x \)-intercept of \( f \), for \( 0 \leq x < 2\pi \). \([1 \text{ mark}]\)

**Markscheme**

\( \frac{3\pi}{2} \) \( \text{A1 N1} \)

Examiners report

Many candidates again had difficulty finding the common angles in the trigonometric equations. In part (a), some did not show sufficient working in solving the equations. Others obtained a single solution in (a)(i) and did not find another. Some candidates worked in degrees; the majority worked in radians.

61b. Write down the exact value of the \( x \)-intercept of \( f \), for \( 0 \leq x < 2\pi \). \([1 \text{ mark}]\)

**Markscheme**

\( \frac{3\pi}{2} \) \( \text{A1 N1} \)

[1 mark]
Examiners report
While some candidates appeared to use their understanding of the graph of the original function to find the x-intercept in part (b), most used their working from part (a)(ii) sometimes with follow-through on an incorrect answer.

61c. The area of the shaded region is \( k \). Find the value of \( k \), giving your answer in terms of \( \pi \). [6 marks]

Markscheme
evidence of using anti-differentiation \((M1)\)
e.g. \( \int_0^{\frac{3\pi}{2}} (6 + 6\sin x) dx \)
correct integral \( 6x - 6\cos x \) (see anywhere) \( A1A1 \)
correct substitution \((A1)\)
e.g. \( 6 \left( \frac{3\pi}{2} \right) - 6\cos \left( \frac{3\pi}{2} \right) - (-6\cos 0) \), \( 9\pi - 0 + 6 \)
\( k = 9\pi + 6 \) \( A1A1 \) \( N3 \)

[6 marks]

Examiners report
Most candidates recognized the need for integration in part (c) but far fewer were able to see the solution through correctly to the end. Some did not show the full substitution of the limits, having incorrectly assumed that evaluating the integral at 0 would be 0; without this working, the mark for evaluating at the limits could not be earned. Again, many candidates had trouble working with the common trigonometric values.

61d. Let \( g(x) = 6 + 6\sin \left( x - \frac{\pi}{2} \right) \). The graph of \( f \) is transformed to the graph of \( g \).

Give a full geometric description of this transformation.

Markscheme
translation of \( \left( \frac{\pi}{2}, 0 \right) \) \( A1A1 \) \( N2 \)

[2 marks]

Examiners report
While there was an issue in the wording of the question with the given domains, this did not appear to bother candidates in part (d). This part was often well completed with candidates using a variety of language to describe the horizontal translation to the right by \( \frac{\pi}{2} \).

61e. Let \( g(x) = 6 + 6\sin \left( x - \frac{\pi}{2} \right) \). The graph of \( f \) is transformed to the graph of \( g \).

Given that \( \int_p^{p + \frac{3\pi}{2}} g(x) dx = k \) and \( 0 \leq p < 2\pi \), write down the two values of \( p \).

Markscheme
recognizing that the area under \( g \) is the same as the shaded region in \( f \) \((M1)\)
\( p = \frac{\pi}{2}, p = 0 \) \( A1A1 \) \( N3 \)

[3 marks]
Examiners report
Most candidates who attempted part (e) realized that the integral was equal to the value that they had found in part (c), but a majority tried to integrate the function $g$ without success. Some candidates used sketches to find one or both values for $p$. The problem in the wording of the question did not appear to have been noticed by candidates in this part either.

Let $f(x) = x \cos x$, for $0 \leq x \leq 6$.

62a. Find $f'(x)$.

Markscheme

evidence of choosing the product rule \( (M1) \)
e.g. $x \times (-\sin x) + 1 \times \cos x$

$f'(x) = \cos x - x \sin x \quad A1A1 \quad N3$

[3 marks]

Examiners report
This problem was well done by most candidates. There were some candidates that struggled to apply the product rule in part (a) and often wrote nonsense like $-x \sin x = -\sin x^2$.

62b. On the grid below, sketch the graph of $y = f'(x)$.

[4 marks]
**Markscheme**

Note: Award A1 for correct domain, $0 \leq x \leq 6$ with endpoints in circles, A1 for approximately correct shape, A1 for local minimum in circle, A1 for local maximum in circle.

Examiners report

In part (b), few candidates were able to sketch the function within the required domain and a large number of candidates did not have their calculator in the correct mode.

The graph of $y = p \cos qx + r$, for $-5 \leq x \leq 14$, is shown below.

There is a minimum point at $(0, -3)$ and a maximum point at $(4, 7)$.

63. Find the value of

   (i) $p$;
   (ii) $q$;
   (iii) $r$.  

[6 marks]
### Markscheme

(i) evidence of finding the amplitude \((MI)\)
e.g. \(\frac{7+3}{2}\), amplitude = 5
\(p = -5\) \(A1\) \(N2\)
(ii) period = 8 \((AI)\)
\(q = 0.785 \left( \frac{2\pi}{8} = \frac{\pi}{4} \right)\) \(A1\) \(N2\)
(iii) \(r = \frac{7-3}{2}\) \((AI)\)
\(r = 2\) \(A1\) \(N2\)

[6 marks]

### Examiners report

Many candidates did not recognize that the value of \(p\) was negative. The value of \(q\) was often interpreted incorrectly as the period but most candidates could find the value of \(r\), the vertical translation.

The diagram below shows a quadrilateral \(ABCD\) with obtuse angles \(\angle ABC\) and \(\angle ADC\).

\[
\begin{align*}
AB &= 5 \text{ cm, } BC = 4 \text{ cm, } CD = 4 \text{ cm, } AD = 4 \text{ cm, } B\hat{A}C = 30^\circ, A\hat{B}C = x^\circ, A\hat{D}C = y^\circ. \\
\end{align*}
\]

64a. Use the cosine rule to show that \(AC = \sqrt{41 - 40\cos x}\). \(1\) mark

### Markscheme

correct substitution \(A1\)
e.g. \(25 + 16 - 40\cos x\), \(5^2 + 4^2 - 2 \times 4 \times 5\cos x\)
\(AC = \sqrt{41 - 40\cos x}\) \(AG\)

[1 mark]

### Examiners report

Many candidates worked comfortably with the sine and cosine rules in part (a) and (b).

64b. Use the sine rule in triangle \(ABC\) to find another expression for \(AC\). \(2\) marks
Markscheme

64c. (i) Hence, find $x$, giving your answer to two decimal places. [6 marks]

(ii) Find $AC$.

Markscheme

(i) evidence of appropriate approach using $AC$  $MI$

e.g. $8\sin x = \sqrt{41 - 40\cos x}$, sketch showing intersection

correct solution $8.682\ldots, 111.317\ldots$  $(A1)$

obtuse value $111.317\ldots$  $(A1)$

$x = 111.32$ to 2 dp (do not accept the radian answer $1.94$ )  $A1$  $N2$

(ii) substituting value of $x$ into either expression for $AC$  $(MI)$

e.g. $AC = 8\sin 111.32$

$AC = 7.45$  $A1$  $N2$

[6 marks]

Examiners report

Equally as many did not take the cue from the word "hence" and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question. A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

64d. (i) Find $y$. [5 marks]

(ii) Hence, or otherwise, find the area of triangle ACD.
**Markscheme**

(i) evidence of choosing cosine rule \( \text{(M1)} \)

\[ \cos B = \frac{a^2 + c^2 - b^2}{2ac} \]

correct substitution \( \text{(A1)} \)

\[ \frac{a^2 + c^2 - 7.45^2}{2ac} = 32 - 32 \cos y, \cos y = -0.734\ldots \]

\[ y = 137 \]

(ii) correct substitution into area formula \( \text{(A1)} \)

correct substitution into area formula \( \text{(A1)} \)

area \( = 5.42 \)

[5 marks]

**Examiners report**

Equally as many did not take the cue from the word “hence” and used an alternate method to solve the problem and thus did not receive full marks. Those who managed to set up an equation, again did not go directly to their GDC but rather engaged in a long, laborious analytical approach that was usually unsuccessful. No matter what values were found in (c) (i) most candidates recovered and earned follow through marks for the remainder of the question. A large number of candidates worked in the wrong mode and rounded prematurely throughout this question often resulting in accuracy penalties.

Let \( f(x) = Ae^{kx} + 3 \). Part of the graph of \( f \) is shown below.

The \( y \)-intercept is at \((0, 13)\).

65a. Show that \( A = 10 \). \[2 \text{ marks}\]

**Markscheme**

substituting \((0, 13)\) into function \( \text{(M1)} \)

\[ 13 = Ae^0 + 3 \]

\[ 13 = A + 3 \]

\[ A = 10 \]

[2 marks]

**Examiners report**

This question was quite well done by a great number of candidates indicating that calculus is a topic that is covered well by most centres. Parts (a) and (b) proved very accessible to many candidates.
65b. Given that \( f(15) = 3.49 \) (correct to 3 significant figures), find the value of \( k \). [3 marks]

**Markscheme**

substituting into \( f(15) = 3.49 \) \( A1 \)

e.g. \( 3.49 = 10e^{15k} + 3, 0.049 = e^{15k} \)

evidence of solving equation \( (M1) \)

e.g. sketch, using \( \ln \)

\( k = -0.201 \) (accept \( \frac{\ln(0.049)}{15} \)) \( A1 \) \( N2 \)

[3 marks]

**Examiners report**

This question was quite well done by a great number of candidates indicating that calculus is a topic that is covered well by most centres. Parts (a) and (b) proved very accessible to many candidates.

65c. (i) Using your value of \( k \), find \( f'(x) \). [5 marks]

(ii) Hence, explain why \( f \) is a decreasing function.

(iii) Write down the equation of the horizontal asymptote of the graph \( f \).

**Markscheme**

(i) \( f(x) = 10e^{-0.201x} + 3 \)

\( f(x) = 10e^{-0.201x} \times -0.201 \) (\( = -2.01e^{-0.201x} \)) \( A1A1A1 \) \( N3 \)

**Note:** Award \( A1 \) for \( 10e^{-0.201x} \), \( A1 \) for \( \times -0.201 \), \( A1 \) for the derivative of 3 is zero.

(ii) valid reason with reference to derivative \( R1 \) \( N1 \)

e.g. \( f'(x) < 0 \), derivative always negative

(iii) \( y = 3 \) \( A1 \) \( N1 \)

[5 marks]

**Examiners report**

The chain rule in part (c) was also carried out well. Few however, recognized the command term “hence” and that \( f'(x) < 0 \) guarantees a decreasing function. A common answer for the equation of the asymptote was to give \( y = 0 \) or \( x = 3 \).

65d. Let \( g(x) = -x^2 + 12x - 24 \). [6 marks]

Find the area enclosed by the graphs of \( f \) and \( g \).

\[ 3.8953... \ 8.6940... \]

\[ \int_{3.89}^{8.69} g(x) - f(x) \, dx = \int_{3.89}^{8.69} \left[ (-x^2 + 12x - 24) - (10e^{-0.201x} + 3) \right] \, dx = 19.5 \]
Examiners report

In part (d), it was again surprising and somewhat disappointing to see how few candidates were able to use their GDC effectively to find the area between curves, often not finding correct limits, and often trying to evaluate the definite integral without the GDC, which led nowhere.

Consider \( f(x) = 2 - x^2 \), for \(-2 \leq x \leq 2\) and \( g(x) = \sin e^x \), for \(-2 \leq x \leq 2\). The graph of \( f \) is given below.

66a. On the diagram above, sketch the graph of \( g \). [3 marks]

**Markscheme**

A1A1A1     N3

66b. Solve \( f(x) = g(x) \). [2 marks]

**Markscheme**

\[
\begin{align*}
x &= -1.32, \quad x = 1.68 \text{ (accept } x = -1.41, \ x = 1.39 \text{ if working in degrees)}
\end{align*}
\]

A1A1     N2

[2 marks]
Examiners report
There were candidates who struggled in vain to solve the equation in part (b) algebraically instead of using a GDC. Those that did use their GDCs to solve the equation frequently gave their answers inaccurately, suggesting that they did not know how to use the "zero" function on their calculator but found a rough solution using the "trace" function.

66c. Write down the set of values of \( x \) such that \( f(x) > g(x) \). [2 marks]

Markscheme
\(-1.32 < x < 1.68 \) (accept \(-1.41 < x < 1.39\) if working in degrees) \( A2 \) \( N2 \) [2 marks]

Examiners report
In part (c) they often gave the correct solution, or obtained follow-through marks on their incorrect results to part (b).

Let \( f(x) = e^x \sin 2x + 10 \), for \( 0 \leq x \leq 4 \). Part of the graph of \( f \) is given below.

![Graph of \( f(x) \)]

There is an \( x \)-intercept at the point A, a local maximum point at M, where \( x = p \) and a local minimum point at N, where \( x = q \).

67a. Write down the \( x \)-coordinate of A. [1 mark]

Markscheme
2.31 \( AI \) \( NI \) [1 mark]

Examiners report
Parts (a) and (b) were generally well answered, the main problem being the accuracy.

67b. Find the value of
(i) \( p \); [2 marks]
(ii) \( q \).
67c. Find \( \int_{p}^{q} f(x) \, dx \). Explain why this is not the area of the shaded region. [3 marks]

**Markscheme**
\[ \int_{p}^{q} f(x) \, dx = 9.96 \quad AI \quad N1 \]
split into two regions, make the area below the \( x \)-axis positive \( R1 \quad N2 \) [3 marks]

**Examiners report**
Many students lacked the calculator skills to successfully complete (6)(c) in that they could not find the value of the definite integral. Some tried to find it by hand. When trying to explain why the integral was not the area, most knew the region under the \( x \)-axis was the cause of the integral not giving the total area, but the explanations were not sufficiently clear. It was often stated that the area below the axis was negative rather than the integral was negative.

The number of bacteria, \( n \), in a dish, after \( t \) minutes is given by \( n = 800e^{0.13t} \).

68a. Find the value of \( n \) when \( t = 0 \). [2 marks]

**Markscheme**
\[ n = 800e^{0} \quad (AI) \]
\[ n = 800 \quad AI \quad N2 \] [2 marks]

**Examiners report**
This question seemed to be challenging for the great majority of the candidates.
Part (a) was generally well answered.

68b. Find the rate at which \( n \) is increasing when \( t = 15 \). [2 marks]

**Markscheme**
evidence of using the derivative \( (MI) \)
\[ n'(15) = 731 \quad AI \quad N2 \] [2 marks]
Examiners report

This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered but in parts (b) and (c) they did not consider that rates of change meant they needed to use differentiation. Most students completely missed or did not understand that the question was asking about the instantaneous rate of change, which resulted in the fact that most of them used the original equation. Some did attempt to find an average rate of change over the time interval, but even fewer attempted to use the derivative.

Of those who did realize to use the derivative in (b), a vast majority calculated it by hand instead of using their GDC feature to evaluate it.

68c. After $k$ minutes, the rate of increase in $n$ is greater than 10000 bacteria per minute. Find the least value of $k$, where $k \in \mathbb{Z}$. [4 marks]

Markscheme

METHOD 1

setting up inequality (accept equation or reverse inequality) $\text{AI}$

e.g. $n'(t) > 10000$

evidence of appropriate approach $\text{MI}$

e.g. sketch, finding derivative

$k = 35.1226 \ldots$ (A1)

least value of $k$ is 36 $\text{A1} \text{ N2}$

METHOD 2

$n'(35) = 9842$, and $n'(36) = 11208$ $\text{A2}$

least value of $k$ is 36 $\text{A2} \text{ N2}$

[4 marks]

Examiners report

This question seemed to be challenging for the great majority of the candidates.

Part (a) was generally well answered but in parts (b) and (c) they did not consider that rates of change meant they needed to use differentiation. Most students completely missed or did not understand that the question was asking about the instantaneous rate of change, which resulted in the fact that most of them used the original equation. Some did attempt to find an average rate of change over the time interval, but even fewer attempted to use the derivative.

Of those who did realize to use the derivative in (b), a vast majority calculated it by hand instead of using their GDC feature to evaluate it.

The inequality for part (c) was sometimes well solved using the original function but many failed to round their answers to the nearest integer.
Consider \( f(x) = x \ln(4 - x^2) \), for \(-2 < x < 2\). The graph of \( f \) is given below.

\[ f(x) = x \ln(4 - x^2) \]

69a. Let \( P \) and \( Q \) be points on the curve of \( f \) where the tangent to the graph of \( f \) is parallel to the \( x \)-axis.

(i) Find the \( x \)-coordinate of \( P \) and of \( Q \).

(ii) Consider \( f(x) = k \). Write down all values of \( k \) for which there are exactly two solutions.

**Markscheme**

(i) \(-1.15, 1.15\) \( A1A1 \) \( N2 \)

(ii) recognizing that it occurs at \( P \) and \( Q \) \( (M1) \)

e.g. \( x = -1.15, x = 1.15 \)

\( k = -1.13, k = 1.13 \) \( A1A1 \) \( N3 \)

[5 marks]

**Examiners report**

Many candidates correctly found the \( x \)-coordinates of \( P \) and \( Q \) in (a)(i) with their GDC. In (a)(ii) some candidates incorrectly interpreted the words “exactly two solutions” as an indication that the discriminant of a quadratic was required. Many failed to realise that the values of \( k \) they were looking for in this question were the \( y \)-coordinates of the points found in (a)(i).

69b. Let \( g(x) = x^3 \ln(4 - x^2) \), for \(-2 < x < 2\).

Show that \( g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2) \).

**Markscheme**

- evidence of choosing the product rule \( (M1) \)
- e.g. \( u \left( v \right)' + v \left( u \right)' \)
- derivative of \( x^3 \) is \( 3x^2 \) \( (AI) \)
- derivative of \( \ln(4 - x^2) \) is \( \frac{-2x}{4-x^2} \) \( (AI) \)
- correct substitution \( A1 \)
- e.g. \( x^3 \times \frac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2 \)
- \( g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2) \) \( AG \) \( N0 \)

[4 marks]
Examiners report
Many candidates were unclear in their application of the product formula in the verifying the given derivative of \( g \). Showing that the derivative was the given expression often received full marks though it was not easy to tell in some cases if that demonstration came through understanding of the product and chain rules or from reasoning backwards from the given result.

69c. Let \( g(x) = x^2 \ln(4 - x^3) \), for \(-2 < x < 2\). [2 marks]
Sketch the graph of \( g' \).

Markscheme

Examiners report
Some candidates drew their graphs of the derivative in (c) on their examination papers despite clear instructions to do their work on separate sheets. Most who tried to plot the graph in (c) did so successfully.

69d. Let \( g(x) = x^3 \ln(4 - x^3) \), for \(-2 < x < 2\). [3 marks]
Consider \( g'(x) = w \). Write down all values of \( w \) for which there are exactly two solutions.

Markscheme

Examiners report
Correct solutions to 10(d) were not often seen.

Let \( f(x) = 2x^3 + 3 \) and \( g(x) = e^{3x} - 2 \).

70a. (i) Find \( g(0) \).

(ii) Find \((f \circ g)(0)\). [5 marks]
**Markscheme**

(i) \( g(0) = e^0 - 2 \)  \((\text{AI})\)
\[= -1 \quad \text{AI} \quad \text{N2} \]

(ii) **METHOD 1**

substituting answer from (i) \((\text{M1})\)

e.g. \((f \circ g)(0) = f(-1)\)

correct substitution \( f(-1) = 2(-1)^3 + 3 \) \((\text{AI})\)

\[f(-1) = 1 \quad \text{AI} \quad \text{N3} \]

**METHOD 2**

attempt to find \((f \circ g)(x)\) \((\text{M1})\)

e.g. \((f \circ g)(x) = f(e^{3x} - 2) = 2(e^{3x} - 2)^3 + 3\)

correct expression for \((f \circ g)(x)\) \((\text{AI})\)

\[e.g. \ 2(e^{3x} - 2)^3 + 3 \]

\[(f \circ g)(0) = 1 \quad \text{AI} \quad \text{N3} \]

\([5 \text{ marks}]\)

**Examiners report**

This question was generally done well, although some students consider \(e^0\) to be 0, losing them a mark.

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70b. Find \( f^{-1}(x) \). \([3 \text{ marks}]\)

**Markscheme**

interchanging \(x\) and \(y\) (seen anywhere) \((\text{M1})\)

\[\text{e.g. } x = 2y^3 + 3\]

attempt to solve \((\text{M1})\)

\[\text{e.g. } y^3 = \frac{x - 3}{2}\]

\[f^{-1}(x) = \sqrt[3]{\frac{x - 3}{2}} \quad \text{AI} \quad \text{N3} \]

\([3 \text{ marks}]\)

**Examiners report**

A few candidates composed in the wrong order. Most found the formula of the inverse correctly, even if in some cases there were errors when trying to isolate \(x\) (or \(y\)). A common incorrect solution found was to find \(y = \sqrt[3]{\frac{x - 3}{2}}\).
The diagram below shows the graph of a function $f(x)$, for $-2 \leq x \leq 4$.

71a. Let $h(x) = f(-x)$. Sketch the graph of $h$ on the grid below. [3 marks]
Examiners report

Part (a) was generally solved correctly. Students had no trouble in deciding what transformation had to be done to the graph, although some confused $f(-x)$ with $-f(x)$.

71b. Let $g(x) = \frac{1}{2} f(x - 1)$. The point $A(3, 2)$ on the graph of $f$ is transformed to the point $P$ on the graph of $g$. Find the coordinates of $P$.

Markscheme

evidence of appropriate approach (M1)
e.g. reference to any horizontal shift and/or stretch factor, $x = 3 + 1$ \(, y = \frac{1}{2} \times 2 \)

P is $(4, 1)$ (accept $x = 4$, $y = 1$) A1A1 A1

Examiners report

Part (b) was generally poorly done. They could not “read” that the transformation shifted the curve 1 unit to the right and stretched it in the $y$-direction with a scale factor of $\frac{1}{2}$. It was often seen that the shift was interpreted, but in the opposite direction. Also, the stretch was applied to both coordinates of the point. Those candidates who answered part (a) incorrectly often had trouble on (b) as well, indicating a difficulty with transformations in general. However, there were also candidates who solved part (a) correctly but could not interpret part (b). This would indicate that it is simpler for them to plot the transformation of an entire function than to find how a particular point is transformed.

Let $f(x) = k \log_2 x$.

72a. Given that $f^{-1}(1) = 8$, find the value of $k$. [3 marks]
**Markscheme**

**METHOD 1**
recognizing that \( f(8) = 1 \) \((MI)\)
e.g. \( 1 = k \log_2 8 \)
recognizing that \( \log_2 8 = 3 \) \((AI)\)
e.g. \( 1 = 3k \)
\( k = \frac{1}{3} \) \(A1 \) \(N2\)

**METHOD 2**
attempt to find the inverse of \( f(x) = k \log_2 x \) \((MI)\)
e.g. \( x = k \log_2 y, y = 2^x \)
substituting 1 and 8 \((MI)\)
e.g. \( 1 = k \log_2 8, 2^\frac{1}{3} = 8 \)
\( k = \frac{1}{\log_2 8} \left( k = \frac{1}{3} \right) \) \(A1 \) \(N2\)

[3 marks]

**Examiners report**

A very poorly done question. Most candidates attempted to find the inverse function for \( f \) and used that to answer parts (a) and (b). Few recognized that the explicit inverse function was not necessary to answer the question.

Although many candidates seem to know that they can find an inverse function by interchanging \( x \) and \( y \), very few were able to actually get the correct inverse. Almost none recognized that if \( f^{-1}(1) = 8 \), then \( f(8) = 1 \). Many thought that the letters “log” could be simply “cancelled out”, leaving the 2 and the 8.

---

72b. Find \( f^{-1} \left( \frac{2}{3} \right) \) .

**Markscheme**

**METHOD 1**
recognizing that \( f(x) = \frac{2}{3} \) \((MI)\)
e.g. \( \frac{2}{3} = \frac{1}{3} \log_2 x \)
\( \log_2 x = 2 \) \((AI)\)
\( f^{-1} \left( \frac{2}{3} \right) = 4 \) (accept \( x = 4 \)) \(A2 \) \(N3\)

**METHOD 2**
attempt to find inverse of \( f(x) = \frac{1}{3} \log_2 x \) \((MI)\)
e.g. interchanging \( x \) and \( y \), substituting \( k = \frac{1}{3} \) into \( y = 2^x \)
correct inverse \((AI)\)
e.g. \( f^{-1}(x) = 2^{3x}, 2^{3x} \)
\( f^{-1} \left( \frac{2}{3} \right) = 4 \) \(A2 \) \(N3\)

[4 marks]
Examiners report
A very poorly done question. Most candidates attempted to find the inverse function for \( f \) and used that to answer parts (a) and (b). Few recognized that the explicit inverse function was not necessary to answer the question.

Although many candidates seem to know that they can find an inverse function by interchanging \( x \) and \( y \), very few were able to actually get the correct inverse. Almost none recognized that if \( f^{-1}(1) = 8 \), then \( f(8) = 1 \). Many thought that the letters “log” could be simply “cancelled out”, leaving the 2 and the 8.

Let \( f(x) = 3 + \frac{20}{x^2 - 4} \), for \( x \neq \pm 2 \). The graph of \( f \) is given below.

![Graph of f(x)](image)

The \( y \)-intercept is at the point A.

73a. (i) Find the coordinates of A. [7 marks]
(ii) Show that \( f'(x) = 0 \) at A.

Markscheme
(i) coordinates of A are \((0, -2)\) A1A1 N2
(ii) derivative of \( x^2 - 4 = 2x \) (seen anywhere) A1
evidence of correct approach M1
e.g. quotient rule, chain rule
finding \( f'(x) \) A2
e.g. \( f'(x) = 20 \times (-1) \times (x^2 - 4)^{-2} \times (2x) \times \frac{(-4)(0) - (20)(2x)}{(x^2 - 4)^2} \)
substituting \( x = 0 \) into \( f'(x) \) (do not accept solving \( f'(x) = 0 \)) M1
at A \( f'(x) = 0 \) A1 A1
AG N0
[7 marks]

Examiners report
Almost all candidates earned the first two marks in part (a) (i), although fewer were able to apply the quotient rule correctly.

73b. Write down the range of \( f \). [2 marks]
Markscheme

**correct** inequalities, \( y \leq -2 \), \( y > 3 \), \( FT \) from (a)(i) and (c)  \( A1A1 \) \( N2 \)

[2 marks]

Examiners report

In (d) even those who knew what the range was had difficulty expressing the inequalities correctly.

Let \( f(x) = \sqrt{x} \). Line \( L \) is the normal to the graph of \( f \) at the point \((4, 2)\).

74. Point \( A \) is the \( x \)-intercept of \( L \). Find the \( x \)-coordinate of \( A \).  

[2 marks]

Markscheme

recognition that \( y = 0 \) at \( A \)  \( (MI) \)

e.g. \(-4x + 18 = 0\)

\( x = \frac{18}{4} \left( = \frac{9}{2} \right) \)  \( AI \) \( N2 \)

[2 marks]

Examiners report

Parts (a) and (b) were well done by most candidates.

A farmer wishes to create a rectangular enclosure, \( ABCD \), of area 525 m\(^2\), as shown below.

![Diagram]

75. The fencing used for side \( AB \) costs $11 per metre. The fencing for the other three sides costs $3 per metre. The farmer creates an enclosure so that the cost is a minimum. Find this minimum cost.  

[7 marks]
**Markscheme**

**METHOD 1**

correct expression for second side, using area = 525 \((A1)\)
e.g. let \(AB = x\), \(AD = \frac{525}{x}\)

attempt to set up cost function using \$3 for three sides and \$11 for one side \((MI)\)
e.g. \(3(AD + BC + CD) + 11AB\)
correct expression for cost \(A2\)
e.g. \(\frac{525}{x} \times 3 + \frac{525}{x} \times 3 + 11x + 3x\), \(\frac{525}{AB} \times 3 + \frac{525}{AB} \times 3 + 11AB + 3AB\), \(\frac{3150}{x} + 14x\)

EITHER

sketch of cost function \((MI)\)

identifying minimum point \((A1)\)
e.g. marking point on graph, \(x = 15\)

minimum cost is 420 (dollars) \(A1\) \(N4\)

OR

correct derivative (may be seen in equation below) \((A1)\)
e.g. \(C'(x) = -\frac{1575}{x^2} + \frac{1575}{x^2} + 14\)

setting their derivative equal to 0 (seen anywhere) \((MI)\)
e.g. \(-\frac{3150}{x^2} + 14 = 0\)

minimum cost is 420 (dollars) \(A1\) \(N4\)

**METHOD 2**

correct expression for second side, using area = 525 \((A1)\)
e.g. let \(AD = x\), \(AB = \frac{525}{x}\)

attempt to set up cost function using \$3 for three sides and \$11 for one side \((MI)\)
e.g. \(3(AD + BC + CD) + 11AB\)
correct expression for cost \(A2\)
e.g. \(3\left(x + x + \frac{525}{x}\right) + \frac{525}{x} \times 11 + 3\left(AD + AD + \frac{525}{AB}\right) + \frac{525}{AD} \times 11 + 6x + \frac{7350}{x}\)

EITHER

sketch of cost function \((MI)\)

identifying minimum point \((A1)\)
e.g. marking point on graph, \(x = 35\)

minimum cost is 420 (dollars) \(A1\) \(N4\)

OR

correct derivative (may be seen in equation below) \((A1)\)
e.g. \(C'(x) = 6 - \frac{7350}{x^2}\)

setting their derivative equal to 0 (seen anywhere) \((MI)\)
e.g. \(6 - \frac{7350}{x^2} = 0\)

minimum cost is 420 (dollars) \(A1\) \(N4\)

[7 marks]
Examiners report
Although this question was a rather straight-forward optimisation question, the lack of structure caused many candidates difficulty. Some were able to calculate cost values but were unable to create an algebraic cost function. Those who were able to create a cost function in two variables often could not use the area relationship to obtain a function in a single variable and so could make no further progress. Of those few who created a correct cost function, most set the derivative to zero to find that the minimum cost occurred at $x = 15$, leading to $\$420$. Although this is a correct approach earning full marks, candidates seem not to recognise that the result can be obtained from the GDC, without the use of calculus.

Let $f(x) = 5 \cos \frac{\pi}{4} x$ and $g(x) = -0.5x^2 + 5x - 8$ for $0 \leq x \leq 9$.

76a. On the same diagram, sketch the graphs of $f$ and $g$. [3 marks]

**Markscheme**

![Graph of f and g]

$AIAIAI$ $N3$

Note: Award $A1$ for $f$ being of sinusoidal shape, with 2 maxima and one minimum, $A1$ for $g$ being a parabola opening down, $A1$ for two intersection points in approximately correct position.

[3 marks]

Examiners report
Graph sketches were much improved over previous sessions. Most candidates graphed the two functions correctly, but many ignored the domain restrictions.

76b. Consider the graph of $g$. Write down

(i) the two $x$-intercepts;
(ii) the equation of the axis of symmetry.

**Markscheme**

(i) $(2, 0), (8, 0)$ (accept $x = 2, x = 8$) $AIAI$ $NINI$
(ii) $x = 5$ (must be an equation) $AI$ $NI$

[3 marks]

Examiners report
Many candidates found parts (b) and (c) accessible, although quite a few did not know how to find the period of the cosine function.
Let $f(x) = x^2$ and $g(x) = 2(x - 1)^2$.

77a. The graph of $g$ can be obtained from the graph of $f$ using two transformations. [2 marks]

Give a full geometric description of each of the two transformations.

**Markscheme**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translated 1 unit to the right</td>
<td>AI NI</td>
</tr>
<tr>
<td>Stretched vertically by factor 2</td>
<td>AI NI</td>
</tr>
</tbody>
</table>

[2 marks]

**Examiners report**
The translation was often described well as horizontal (or shift) one unit right. There was considerable difficulty describing the vertical stretch as it was often referred to as "stretch by 2" or "amplitude of 2". A full description should include the name (e.g. vertical stretch) and value for full marks. Candidates also had difficulty applying two consecutive transformations to a single point.

Often the translations were applied directly to $(-1, 1)$ instead of first mapping from $f$ to $g$. A good number of candidates correctly found $h(x)$, but most could not find $P$ from this function.

77b. The graph of $g$ is translated by the vector \[ \begin{pmatrix} 3 \\ -2 \end{pmatrix} \] to give the graph of $h$. [4 marks]

The point $(-1, 1)$ on the graph of $f$ is translated to the point $P$ on the graph of $h$.

Find the coordinates of $P$.

**Markscheme**

**METHOD 1**

Finding coordinates of image on $g$ (AI)(AI)

e.g. $-1 + 1 = 0, 1 \times 2 = 2, (-1, 1) \rightarrow (-1 + 1, 2 \times 1) = (0, 2)$

P is $(3, 0)$ AIAI N4

**METHOD 2**

$h(x) = 2(x - 4)^2 - 2$ (AI)(AI)

P is $(3, 0)$ AIAI N4

**Examiners report**
The translation was often described well as horizontal (or shift) one unit right. There was considerable difficulty describing the vertical stretch as it was often referred to as "stretch by 2" or "amplitude of 2". A full description should include the name (e.g. vertical stretch) and value for full marks. Candidates also had difficulty applying two consecutive transformations to a single point.

Often the translations were applied directly to $(-1, 1)$ instead of first mapping from $f$ to $g$. A good number of candidates correctly found $h(x)$, but most could not find $P$ from this function.

Let $f(x) = e^{x+3}$.

78. (i) Show that $f^{-1}(x) = \ln x - 3$. [3 marks]

(ii) Write down the domain of $f^{-1}$. 

**Markscheme**

(i) interchanging $x$ and $y$ (seen anywhere)  \( M1 \)

correct manipulation  \( A1 \)

e.g. $\ln x = y + 3$, $\ln y = x + 3$

\( f^{-1}(x) = \ln x - 3 \)  \( AG \)  \( N0 \)

(ii) $x > 0$  \( A1 \)  \( N1 \)

[3 marks]

**Examiners report**

Many candidates interchanged the $x$ and $y$ to find the inverse function, but very few could write down the correct domain of the inverse, often giving $x \geq 0$, $x > 3$ and "all real numbers" as responses.

Let $f(x) = \frac{ax}{x^2 + 1}$, $-8 \leq x \leq 8$, $a \in \mathbb{R}$. The graph of $f$ is shown below.

The region between $x = 3$ and $x = 7$ is shaded.

79a. Show that $f(-x) = -f(x)$.  \( [2 \text{ marks}] \)

**Markscheme**

**METHOD 1**

evidence of substituting $-x$ for $x$  \( M1 \)

\[ f(-x) = \frac{-ax}{(-x)^2 + 1} \]  \( A1 \)

\[ f(-x) = -\frac{ax}{x^2 + 1} = -f(x) \]  \( AG \)  \( N0 \)

**METHOD 2**

$y = -f(x)$ is reflection of $y = f(x)$ in $x$ axis

and $y = f(-x)$ is reflection of $y = f(x)$ in $y$ axis  \( M1 \)

sketch showing these are the same  \( A1 \)

\[ f(-x) = -\frac{ax}{x^2 + 1} = -f(x) \]  \( AG \)  \( N0 \)

[2 marks]


Examiners report

Part (a) was achieved by some candidates, although brackets around the \(-x\) were commonly neglected. Some attempted to show the relationship by substituting a specific value for \(x\). This earned no marks as a general argument is required.

79b. It is given that \(f(x) = \frac{5}{2} \ln(x^2 + 1) + C\). 

(i) Find the area of the shaded region, giving your answer in the form \(p \ln q\).

(ii) Find the value of \(\int^{8}_4 2f(x - 1)\, dx\).

Markscheme

(i) correct expression \(A2\)

e.g. \(\left[\frac{5}{2} \ln(x^2 + 1)\right]_3^7, \frac{5}{2} \ln 50 - \frac{5}{2} \ln 10, \frac{5}{2} (\ln 50 - \ln 10)\)

area = \(\frac{5}{2} \ln 5\) \(A1A1\) \(N2\)

(ii) METHOD 1

recognizing the shift that does not change the area \((MI)\)

e.g. \(\int^{8}_4 f(x - 1)\, dx = \int^{7}_3 f(x)\, dx, \frac{5}{2} \ln 5\)

METHOD 2

changing variable

let \(w = x - 1\), so \(\frac{dw}{dx} = 1\)

\(2 \int f(w)\, dw = \frac{2w}{2} \ln(w^2 + 1) + c\) \((MI)\)

substituting correct limits

e.g. \(\left[\ln(w^2 + 1)\right]_3^7, \ln 50 - \ln 10\) \((MI)\)

\(\int^{8}_4 2f(x - 1)\, dx = a \ln 5\) (i.e. 2 \times their answer to (c)(i)) \(A1\) \(N3\)

[7 marks]

Examiners report

For those who found a correct expression in (c)(i), many finished by calculating \(\ln 50 - \ln 10 = \ln 40\). Few recognized that the translation did not change the area, although some factored the 2 from the integrand, appreciating that the area is double that in (c)(i).

Let \(f(x) = \frac{3x}{2} + 1\), \(g(x) = 4 \cos \left(\frac{x}{3}\right) - 1\). Let \(h(x) = (g \circ f)(x)\).

80. Find an expression for \(h(x)\). 

[3 marks]
Markscheme

attempt to form any composition (even if order is reversed) \( (M1) \)

correct composition \( h(x) = g \left( \frac{3x + 2}{6} \right) + 1 \) \( (A1) \)

\[
h(x) = 4 \cos \left( \frac{3x + 2}{6} \right) - 1 - \left( 4 \cos \left( \frac{1}{2}x + \frac{1}{3} \right) - 1, 4 \cos \left( \frac{3x + 2}{6} \right) - 1 \right) \quad A1 \quad N3
\]

[3 marks]

Examiners report

The majority of candidates handled the composition of the two given functions well. However, a large number of candidates had difficulties simplifying the result correctly. The period and range of the resulting trig function was not handled well. If candidates knew the definition of "range", they often did not express it correctly. Many candidates correctly used their GDCs to find the period and range, but this approach was not the most efficient.

In a geometric series, \( u_1 = \frac{1}{81} \) and \( u_4 = \frac{1}{3} \).

81. Find the smallest value of \( n \) for which \( S_n > 40 \). \quad [4 marks]

Markscheme

METHOD 1

setting up an inequality (accept an equation) \( M1 \)

e.g. \( \frac{1}{2} (3^n - 1) > 40, \frac{1}{2} (3^n - 1) > 40, 3^n > 6481 \)
evidence of solving \( M1 \)
e.g. graph, taking logs

\[
n > 7.9888... \quad (A1)
\]

\[\therefore n = 8 \quad A1 \quad N2\]

METHOD 2

if \( n = 7 \), sum = 13.49...; if \( n = 8 \), sum = 40.49... \quad A2

\( n = 8 \) (is the smallest value) \quad A2 \quad N2

[4 marks]

Examiners report

In part (b) a good number of candidates did not realize that they could use logs to solve the problem, nor did they make good use of their GDCs. Some students did use a trial and error approach to check various values however, in many cases, they only checked one of the "crossover" values. Most candidates had difficulty with notation, opting to set up an equation rather than an inequality.

Let \( f(x) = x^3 - 4x + 1 \).

82. Write down the range of values for the gradient of \( f \). \quad [2 marks]

Markscheme

\( f'(x) \geq -4, y \geq -4, \quad [-4, \infty[ \quad A2 \quad N2 \)

[2 marks]
Examiners report
Part (e) was often not attempted and if it was, candidates were not clear on what was expected.

Let \( f(x) = x^2 \) and \( g(x) = 2x - 3 \).

83a. Find \( g^{-1}(x) \).

**Markscheme**
for interchanging \( x \) and \( y \) (may be done later) \( (MI) \)
e.g. \( x = 2y - 3 \)
\( g^{-1}(x) = \frac{x+3}{2} \) (accept \( y = \frac{x+3}{2}, \frac{x+3}{2} \)) \( AI \) \( N2 \)

[2 marks]

Examiners report
Many candidates performed successfully in finding the inverse function, as well as the composite at a specified value of \( x \).

83b. Find \( (f \circ g)(4) \).

**Markscheme**
METHOD 1
\( g(4) = 5 \) \( (AI) \)
evidence of composition of functions \( (MI) \)
\( f(5) = 25 \) \( AI \) \( N3 \)

METHOD 2
\( f \circ g(x) = (2x - 3)^2 \) \( (MI) \)
\( f \circ g(4) = (2 \times 4 - 3)^2 \) \( (AI) \)
\( = 25 \) \( AI \) \( N3 \)

[3 marks]

Examiners report
Many candidates performed successfully in finding the inverse function, as well as the composite at a specified value of \( x \). Some candidates made arithmetical errors especially if they expanded the binomial before substituting \( x = 4 \).
Consider the graph of $f$ shown below.

84a. On the same grid sketch the graph of $y = f(-x)$.

**Markscheme**

Examiners report

This question was reasonably solved by many students, though a good number confused $f(-x)$ with $-f(x)$ in part (a), thus reflecting the original diagram in the $x$-axis. Candidates need more practice in correctly and fully describing transformations.
The following four diagrams show images of \( f \) under different transformations.

84b. Complete the following table. [2 marks]

<table>
<thead>
<tr>
<th>Description of transformation</th>
<th>Diagram letter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal stretch with scale factor 1.5</td>
<td>C</td>
</tr>
<tr>
<td>Maps ( f ) to ( f(x) + 1 )</td>
<td>D</td>
</tr>
</tbody>
</table>

Markscheme

<table>
<thead>
<tr>
<th>Description of transformation</th>
<th>Diagram letter</th>
<th>A1</th>
<th>A1</th>
<th>N2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal stretch with scale factor 1.5</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maps ( f ) to ( f(x) + 1 )</td>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Examiners report

Candidates need more practice in correctly and fully describing transformations. There was often confusion between the description of the transformation and the equation that represents it. A fairly low percentage of the candidates used the term "translation".

84c. Give a full geometric description of the transformation that gives the image in Diagram A. [2 marks]

Markscheme

translation (accept move/shift/slide etc.) with vector \( \begin{pmatrix} -6 \\ -2 \end{pmatrix} \) A1 A1 N2

Examiners report

Candidates need more practice in correctly and fully describing transformations. There was often confusion between the description of the transformation and the equation that represents it. A fairly low percentage of the candidates used the term "translation".
85. Solve the equation $e^x = 4 \sin x$, for $0 \leq x \leq 2\pi$.

**Markscheme**

evidence of appropriate approach  **M1**
e.g. a sketch, writing $e^x - 4 \sin x = 0$

$x = 0.371, x = 1.36$  **A2**  **N2**

**Examiners report**

Although many students started with an analytical approach, many also realized they were not going further and successfully used their GDC to find the intercepts with the x-axis if they had set the equation equal to 0, or in other cases, they found the intersection of the two graphs. The better candidates drew a reasonable sketch and found the two values without difficulty. A good number of candidates did not provide a sketch, however, and they had more trouble earning the mark for showing method. Accuracy penalties were relatively common on this question.

The quadratic equation $kx^2 + (k - 3)x + 1 = 0$ has two equal real roots.

86a. Find the possible values of $k$.

**Markscheme**

attempt to use discriminant  **(M1)**
correct substitution, $(k - 3)^2 - 4 	imes k 	imes 1$  **(A1)**
setting their discriminant equal to zero  **M1**
e.g. $(k - 3)^2 - 4 	imes k 	imes 1 = 0$, $k^2 - 10k + 9 = 0$

$k = 1, k = 9$  **A1**  **N3**

**Examiners report**

Although some candidates correctly considered the discriminant to find the possible values of $k$, many of them did not set it equal to 0, writing an inequality instead.

86b. **Write down** the values of $k$ for which $x^2 + (k - 3)x + k = 0$ has two equal real roots.

**Markscheme**

$k = 1, k = 9$  **A2**  **N2**

**Examiners report**

In part (b), some students realized that the discriminants in parts (a) and (b) were the same, earning follow through marks just by writing the same (often incorrect) answers they got in part (a). Many, however, did not see the connection between the two parts.

Let $f(x) = 3 \sin x + 4 \cos x$, for $-2\pi \leq x \leq 2\pi$.

87a. Sketch the graph of $f$.

**Markscheme**

[3 marks]
**Markscheme**

![Graph Image]

*Note: Award A1 for approximately sinusoidal shape, A1 for end points approximately correct \((-2\pi, 4) (2\pi, 4)\), A1 for approximately correct position of graph, (y-intercept \((0, 4)\), maximum to right of y-axis).*

**[3 marks]**

**Examiners report**

Some graphs in part (a) were almost too detailed for just a sketch but more often, the important features were far from clear. Some graphs lacked scales on the axes.

---

87b. Write down **[3 marks]**

(i) the amplitude;

(ii) the period;

(iii) the x-intercept that lies between \(-\frac{\pi}{2}\) and 0.

**Markscheme**

(i) 5 **A1 N1**

(ii) \(2\pi\) (6.28) **A1 N1**

(iii) \(-0.927\) **A1 N1**

**[3 marks]**

**Examiners report**

A number of candidates had difficulty finding the period in part (b)(ii).

---

87c. Let \(g(x) = \ln(x + 1)\), for \(0 \leq x \leq \pi\). There is a value of \(x\), between 0 and 1, for which the gradient of \(f\) is equal to the gradient of \(g\). Find this value of \(x\). **[5 marks]**
Markscheme

METHOD 1

graphical approach (but must involve derivative functions)  \textit{MI}

e.g.

\[ e.g. \]

<table>
<thead>
<tr>
<th>A1</th>
<th>A1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2</td>
<td>N2</td>
</tr>
</tbody>
</table>

METHOD 2

g'(x) = \frac{1}{x+1}  \textit{AI}

f'(x) = 3\cos x - 4\sin x - (5\cos(x + 0.927))  \textit{AI}

evidence of attempt to solve g'(x) = f'(x)  \textit{MI}

| x = 0.511 | A2 | N2 |

[5 marks]

Examiners report

In part (f), many candidates were able to get as far as equating the two derivatives but fewer used their GDC to solve the resulting equation. Again, many had trouble demonstrating their method of solution.

Let \( f \) be the function given by \( f(x) = e^{0.5x} \), \( 0 \leq x \leq 3.5 \). The diagram shows the graph of \( f \).

88a. On the same diagram, sketch the graph of \( f^{-1} \).  \[3 \text{ marks}\]
88b. Write down the range of $f^{-1}$.

[1 mark]

Markscheme

$0 \leq y \leq 3.5$  
$A1$  
$N1$  
[1 mark]

Examiners report

There were a large number of candidates who were unaware of the geometric relationship between a function and its inverse. Those that had some idea of the shape of the graph often did not consider the specified domain. Many more students were able to use an analytical approach to finding the inverse of a function and had little problem using logarithms to solve for $y$. Candidates were clearly more comfortable with algebraic procedures than graphical interpretations.

88c. Find $f^{-1}(x)$.

[3 marks]

Markscheme

interchanging $x$ and $y$ (seen anywhere)  
$MI$

e.g. $x = e^{0.5y}$  
$AI$

evidence of changing to log form  
$AI$

e.g. $\ln x = 0.5y$, $\ln x = \ln e^{0.5y}$ (any base), $\ln x = 0.5y \ln e$ (any base)

$f^{-1}(x) = 2\ln x$  
$A1$  
$N1$  
[3 marks]
Examiners report
There were a large number of candidates who were unaware of the geometric relationship between a function and its inverse. Those that had some idea of the shape of the graph often did not consider the specified domain. Many more students were able to use an analytical approach to finding the inverse of a function and had little problem using logarithms to solve for $y$. Candidates were clearly more comfortable with algebraic procedures than graphical interpretations.

Let $f(t) = a \cos b(t - c) + d, t \geq 0$. Part of the graph of $y = f(t)$ is given below.

![Graph](image)

When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

89a. The transformation $P$ is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $(\begin{array}{c}3 \\ -10\end{array})$. Let $M'$ be the image of M under $P$. Find the coordinates of $M'$.  

[2 marks]

**Markscheme**

stretch takes 3 to 1.5 ($A1$)  
translation maps $(1.5, 29)$ to $(4.5, 19)$ (so $M'$ is $(4.5, 19)$) $A1$ $N2$  
[2 marks]

**Examiners report**

Few answered part (b) correctly as most could not interpret the horizontal stretch.

89b. The graph of $g$ is the image of the graph of $f$ under $P$. Find $g(t)$ in the form $g(t) = 7 \cos B(t - c) + D$.  

[4 marks]

**Markscheme**

$g(t) = 7 \cos \frac{\pi}{3} (t - 4.5) + 12$ $A1A2A1$ $N4$  

Note: Award $A1$ for $\frac{\pi}{3}$, $A2$ for 4.5, $A1$ for 12.

Other correct values for $c$ can be found, $c = 4.5 \pm 6k, k \in \mathbb{Z}$.  

[4 marks]
Examiners report

Few answered part (b) correctly as most could not interpret the horizontal stretch. As a result, there were many who were unable to answer part (c) although follow through marks were often obtained from incorrect answers in both parts (a) and (b). The link between the answer in (b) and the value of $C$ in part (c) was lost on all but the most attentive.

89c. The graph of $g$ is the image of the graph of $f$ under $P$. Give a full geometric description of the transformation that maps the graph of $g$ to the graph of $f$.

**Markscheme**

- translation $\left( \begin{array}{l} -3 \\ 10 \end{array} \right)$ (AI)
- horizontal stretch of a scale factor of 2 (AI)
- completely correct description, in correct order A1 N3
- e.g. translation $\left( \begin{array}{l} -3 \\ 10 \end{array} \right)$ then horizontal stretch of a scale factor of 2 [3 marks]

Examiners report

In part (d), some candidates could name the transformations required, although only a handful provided the correct order of the transformations to return the graph to its original state.

Let $f(x) = 2x^2 + 4x - 6$.

90a. Express $f(x)$ in the form $f(x) = 2(x - h)^2 + k$. [3 marks]

**Markscheme**

- evidence of obtaining the vertex (MI)
- e.g. a graph, $x = -\frac{b}{2a}$, completing the square
- $f(x) = 2(x + 1)^2 - 8$ A2 N3 [3 marks]

Examiners report

Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the $h$ and $k$ values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

90b. Write down the equation of the axis of symmetry of the graph of $f$. [1 mark]

**Markscheme**

- $x = -1$ (equation must be seen) A1 NI [1 mark]
Examiners report
Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the $h$ and $k$ values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

90c. Express $f(x)$ in the form $f(x) = 2(x - p)(x - q)$.

**Markscheme**

$f(x) = 2(x - 1)(x + 3)$  \textit{A1A1 N2}

[2 marks]

Examiners report
Many candidates answered this question with great ease. Still, some found themselves unable to correctly find the vertex algebraically, often mixing the signs of the $h$ and $k$ values. Using the GDC may have been a more fruitful approach. Some candidates did not write the axis of symmetry as an equation.

Let $f(x) = x \cos(x - \sin x)$, $0 \leq x \leq 3$.

91a. Sketch the graph of $f$ on the following set of axes.

**Markscheme**

[3 marks]

Notes: Award A1 for correct domain, $0 \leq x \leq 3$. Award A2 for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2. [3 marks]
**Examiners report**

Many candidates sketched a clear and smooth freehand curve with the local maximum, x-intercept and endpoints in approximately correct positions. Commonly, candidates sketched a graph across $[-3, 3]$, which neglects the given domain of the function. There were some candidates who sketched a straight line through the origin, presumably from being in the degree mode of their GDC.

---

91b. The graph of $f$ intersects the x-axis when $x = a$, $a \neq 0$. Write down the value of $a$.  

**Markscheme**

$a = 2.31$  

1 mark

---

92a. Find $f^{-1}(x)$.  

**Markscheme**

METHOD 1  

$\ln(x + 5) + \ln 2 = \ln(2(x + 5)) = \ln(2x + 10)$  

interchanging $x$ and $y$ (seen anywhere)  

e.g. $x = \ln(2y + 10)$  

evidence of correct manipulation  

e.g. $e^x = 2y + 10$  

$f^{-1}(x) = \frac{e^x - 10}{2}  

AI \ N2$  

METHOD 2  

$y = \ln(x + 5) + \ln 2$  

$y - \ln 2 = \ln(x + 5)$  

evidence of correct manipulation  

e.g. $e^{y - \ln 2} = x + 5$  

interchanging $x$ and $y$ (seen anywhere)  

e.g. $e^{x - \ln 2} = y + 5$  

$f^{-1}(x) = e^{x - \ln 2} - 5  

AI \ N2$  

4 marks

---

**Examiners report**

This was one of the more difficult problems for the candidates. Knowledge of the laws of logarithms appeared weak as did the inverse nature of the exponential and logarithmic functions. There were a number of candidates who mistook the notation for the inverse to mean either the derivative or the reciprocal. The order of composition seemed well understood by most candidates but they were unable to simplify by the rules of indices to obtain the correct final answer.
92b. Let \( g(x) = e^x \).

Find \((g \circ f)(x)\), giving your answer in the form \(ax + b\), where \(a, b \in \mathbb{Z}\).

**Markscheme**

**METHOD 1**

evidence of composition in correct order \((MI)\)
e.g. \( (g \circ f)(x) = g(\ln(x + 5) + \ln 2) \)
\(= e^{\ln(2(x + 5))} = 2(x + 5) \)
\( (g \circ f)(x) = 2x + 10 \) \(A1\) \(N2\)

**METHOD 2**

evidence of composition in correct order \((MI)\)
e.g. \( (g \circ f)(x) = e^{\ln(2(x + 5))} \cdot \ln 2 \)
\(= e^{\ln(2(x + 5))} \cdot \ln 2 = (x + 5)2 \)
\( (g \circ f)(x) = 2x + 10 \) \(A1\) \(N2\)

[3 marks]

**Examiners report**

This was one of the more difficult problems for the candidates. Knowledge of the laws of logarithms appeared weak as did the inverse nature of the exponential and logarithmic functions. There were a number of candidates who mistook the notation for the inverse to mean either the derivative or the reciprocal. The order of composition seemed well understood by most candidates but they were unable to simplify by the rules of indices to obtain the correct final answer.

Let \( f(x) = (x + 1)^2 - 12 \).

93a. Show that \( f(x) = 3x^2 + 6x - 9 \).

**Markscheme**

\( f(x) = 3(x^2 + 2x + 1) - 12 \) \(A1\)
\(= 3x^2 + 6x + 3 - 12 \) \(A1\)
\(= 3x^2 + 6x - 9 \) \(AG\) \(N0\)

[2 marks]

**Examiners report**

This problem was generally well done. The “show that” question in part (a) was done correctly by most candidates, with a few attempting to show it by working backwards, which earned no marks.

93b. For the graph of \( f \)

(i) write down the coordinates of the vertex;
(ii) write down the equation of the axis of symmetry;
(iii) write down the \( y \)-intercept;
(iv) find both \( x \)-intercepts.
**Markscheme**

(i) vertex is \((-1, -12)\)  \(A1\) **N2**

(ii) \(x = -1\) (must be an equation)  \(A1\) **NI**

(iii) \((0, -9)\)  \(A1\) **NI**

(iv) evidence of solving \(f(x) = 0\)  \(M1\)

e.g. factorizing, formula, correct working  \(A1\)

e.g. \(3(x + 3)(x - 1) = 0\), \(x = \frac{-6 \pm \sqrt{36 + 36}}{6}\)

\((-3, 0), (1, 0)\)  \(A1\) **N1**

[8 marks]

**Examiners report**

Most candidates were able to identify the vertex but were unable to write the equation for the axis of symmetry. There was a great deal of success with the \(x\) and \(y\) intercepts.

93c. **Hence** sketch the graph of \(f\).  \([2\,\text{marks}]\)

**Markscheme**

\[\text{Note: Award } A1\text{ for a parabola opening upward, } A1\text{ for vertex and intercepts in approximately correct positions.}\]

[2 marks]

**Examiners report**

Some of the sketches of the graph left much to be desired even if they were technically correct; many were v-shaped.

93d. Let \(g(x) = x^2\). The graph of \(f\) may be obtained from the graph of \(g\) by the two transformations: \([3\,\text{marks}]\)

- a stretch of scale factor \(t\) in the \(y\)-direction

followed by a translation of \(\begin{pmatrix} p \\ q \end{pmatrix}\).

Find \(\begin{pmatrix} p \\ q \end{pmatrix}\) and the value of \(t\).
Markscheme

\[
\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}, \quad t = 3 \text{ (accept } p = -1, q = -12, t = 3 \text{ )} \quad \text{AIAIAI N3}
\]

[3 marks]

Examiners report

The final part was poorly done, indicating that defining a graph in terms of stretch and translation was unfamiliar to many candidates.

Let \( f(x) = 4 \tan^2 x - 4 \sin x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3} \).

94a. On the grid below, sketch the graph of \( y = f(x) \). [3 marks]

![Graph Sketch](image)

Markscheme

Note: Award A1 for passing through (0, 0), A1 for correct shape, A1 for a range of approximately -1 to 15.

[3 marks]

Examiners report

In part (a), some did not realize that they should copy the curve from their GDC, paying attention to domain and range.
94b. Solve the equation \( f(x) = 1 \).

**Markscheme**

- evidence of attempt to solve \( f(x) = 1 \) \( (M1) \)
  - e.g. line on sketch, using \( \tan x = \frac{\sin x}{\cos x} \)
  - \( x = -0.207, x = 0.772 \) \( A1A1 \ N3 \)

**Examiners report**

Not using their GDC, and trying to solve the equation analytically in part (b) proved to be very difficult for many. A common error was to substitute \( x = 1 \).

The following diagram shows the graphs of \( f(x) = \ln(3x - 2) + 1 \) and \( g(x) = -4 \cos(0.5x) + 2 \), for \( 1 \leq x \leq 10 \).

95. There are two values of \( x \) for which the gradient of \( f \) is equal to the gradient of \( g \). Find both these values of \( x \).

**Markscheme**

- evidence of using derivatives for gradients \( (M1) \)
  - correct approach \( (A1) \)
  - e.g. \( f'(x) = g'(x) \), points of intersection
  - \( x = 1.43, x = 6.10 \) \( A1A1 \ N2N2 \)

**Examiners report**

Most candidates realized the relationship between the gradient and the first derivative and set the two derivatives equal to one another. Once again many did not realize that the intersection could be easily found on their GDC.
The following diagram shows part of the graph of \( f \), where \( f(x) = x^2 - x - 2 \).

96a. Find both \( x \)-intercepts. \[4 \text{ marks}\]

**Markscheme**
- evidence of attempting to solve \( f(x) = 0 \) (\( M1 \))
- evidence of correct working (\( A1 \))
- e.g. \((x + 1)(x - 2)\), \( \frac{1 \pm \sqrt{3}}{2} \)
- intercepts are \((-1, 0)\) and \((2, 0)\) (accept \( x = -1, x = 2 \)) \( A1 A1 \) \( N1 N1 \)

\[4 \text{ marks}\]

**Examiners report**
This question was consistently the best handled one on the entire paper.

96b. Find the \( x \)-coordinate of the vertex. \[2 \text{ marks}\]

**Markscheme**
- evidence of appropriate method (\( M1 \))
- e.g. \( x_v = \frac{x_1 + x_2}{2} \), \( x_v = -\frac{b}{2a} \), reference to symmetry
- \( x_v = 0.5 \) \( A1 \) \( N2 \)

\[2 \text{ marks}\]

**Examiners report**
This question was consistently the best handled one on the entire paper.
Part of the graph of a function $f$ is shown in the diagram below.

97a. On the same diagram sketch the graph of $y = -f(x)$.

**Markscheme**

**M1A1 N2**

Note: Award M1 for evidence of reflection in $x$-axis, A1 for correct vertex and all intercepts approximately correct.

**Examiners report**

This question was reasonably well done. Many recognized the graph of $-f(x)$ as a reflection in a horizontal line, but fewer recognized the $x$-axis as the mirror line.

97b. Let $g(x) = f(x + 3)$.

(i) Find $g(-3)$.

(ii) Describe fully the transformation that maps the graph of $f$ to the graph of $g$. 

[4 marks]
Markscheme

(i) \( g(-3) = f(0) \) \((A1)\)
\( f(0) = -1.5 \) \((AI)\) \((N2)\)

(ii) translation (accept shift, slide, etc.) of \( \begin{pmatrix} -3 \\ 0 \end{pmatrix} \) \((AI)\) \((N2)\)

\([4\text{ marks}]\)

Examiners report

A fair number gave \( g(-3) = f(0) \), but did not carry through to \( f(0) = -1.5 \). The majority of candidates recognized that moving the graph of \( f(x) \) by 3 units to the left results in the graph of \( g(x) \), but the language used to describe the transformation was often far from precise mathematically.

Let \( f(x) = e^{x(1-x^2)} \).

Part of the graph of \( y = f(x) \), for \(-6 \leq x \leq 2\), is shown below. The x-coordinates of the local minimum and maximum points are \( r \) and \( s \) respectively.

98. Write down the equation of the horizontal asymptote. \([1\text{ mark}]\)

Markscheme

\( y = 0 \) \((AI)\) \((N1)\)

\([1\text{ mark}]\)

Examiners report

For part (b), the equation of the horizontal asymptote was commonly written as \( x = 0 \).

A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After \( n \) years the number of taxis, \( T \), in the city is given by

\[ T = 280 \times 1.12^n. \]

99a. (i) Find the number of taxis in the city at the end of 2005. \([6\text{ marks}]\)

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.
Markscheme

(i) \( n = 5 \) (A1)
\[ T = 280 \times 1.12^5 \]
\[ T = 493 \quad A1 \quad N2 \]

(ii) evidence of doubling (A1)

e.g. 560

setting up equation A1

e.g. \( 280 \times 1.12^n = 560, 1.12^n = 2 \)

\( n = 6.116 \ldots \) (A1)

in the year 2007 A1 N3

[6 marks]

Examiners report

A number of candidates found this question very accessible. In part (a), many correctly solved for \( n \), but often incorrectly answered with the year 2006, thus misinterpreting that 6.12 years after the end of 2000 is in the year 2007.

99b. At the end of 2000 there were 25600 people in the city who used taxis. [6 marks]

After \( n \) years the number of people, \( P \), in the city who used taxis is given by

\[ P = \frac{256000}{10 + 90e^{-0.1n}}. \]

(i) Find the value of \( P \) at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of \( P \) be double its value at the end of 2000? Justify your answer.

Markscheme

(i) \( P = \frac{256000}{10 + 90e^{-0.1n}} \) (A1)
\[ P = 39635.993 \ldots \] (A1)
\[ P = 39636 \quad A1 \quad N3 \]

(ii) \( P = \frac{256000}{10 + 90e^{-0.1n}} \)
\[ P = 46806.997 \ldots \] A1

not doubled A1 N0

valid reason for their answer R1

e.g. \( P < 51200 \)

[6 marks]

Examiners report

Many found correct values in part (b) and often justified their result by simply noting the value after seven years is less than 51200. A common alternative was to divide 46807 by 25600 and note that this ratio is less than two. There were still a good number of candidates who failed to provide any justification as instructed.

99c. Let \( R \) be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if \( R < 70 \). [5 marks]

(i) Find the value of \( R \) at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?
**Markscheme**

(i) correct value \( A2 \quad N2 \)

\[ \frac{25600}{280^2} \cdot 91.4, 640 : 7 \]

(ii) setting up an inequality (accept an equation, or reversed inequality) \( M1 \)

\[ \frac{P}{T} < 70, \quad \frac{25600}{(10+90e^{-0.1})} < 70 \]

finding the value 9.31... \( (A1) \)

after 10 years \( A1 \quad N2 \)

[5 marks]

**Examiners report**

Part (c) proved more challenging to candidates. Many found the correct ratio for \( R \), however few candidates then created a proper equation or inequality by dividing the function for \( P \) by the function for \( T \) and setting this equal (or less) than 70. Such a function, although unfamiliar, can be solved using the graphing or solving features of the GDC. Many candidates chose a tabular approach but often only wrote down one value of the table, such as \( n = 10, R = 68.3 \). What is essential is to include the two values between which the correct answer falls. Sufficient evidence would include \( n = 9, R = 70.8 \) so that it is clear the value of \( R = 70 \) has been surpassed.

Let \( f(x) = 3(x + 1)^2 - 12 \).

100a. Show that \( f(x) = 3x^2 + 6x - 9 \).

[2 marks]

**Markscheme**

\[ f(x) = 3(x^2 + 2x + 1) - 12 \quad A1 \]

\[ = 3x^2 + 6x + 12 \quad A1 \]

\[ = 3x^2 + 6x - 9 \quad AG \quad N0 \]

[2 marks]

**Examiners report**

[N/A]

100b. For the graph of \( f \)

(i) write down the coordinates of the vertex;

(ii) write down the \( y \)-intercept;

(iii) find both \( x \)-intercepts.

[7 marks]
**Markscheme**

(i) vertex is \((-1, -12)\)  
A1A1 N2

(ii) \(y = -9\) , or \((0, -9)\)  
A1 NI

(iii) evidence of solving \(f(x) = 0\)  
M1  
e.g. factorizing, formula  
correct working  
A1  
e.g.  
\[3(x + 3)(x - 1) = 0, \quad x = \frac{-6 \pm \sqrt{54}}{6}\]  
\(x = -3, \quad x = 1\) , or \((-3, 0), (1, 0)\)  
A1A1 N2  
[7 marks]

**Examiners report**

[N/A]

100c. **Hence** sketch the graph of \(f\).  
[3 marks]

**Markscheme**

![Graph](image)

A1A1AI N3

**Note:** Award A1 for a parabola opening upward, A1 for vertex in approximately correct position, A1 for intercepts in approximately correct positions. Scale and labelling not required.  
[3 marks]

**Examiners report**

[N/A]

100d. Let \(g(x) = x^2\) . The graph of \(f\) may be obtained from the graph of \(g\) by the following two transformations  
[3 marks]

a stretch of scale factor \(t\) in the \(y\)-direction,  
followed by a translation of \(\begin{pmatrix} p \\ q \end{pmatrix}\).

Write down \(\begin{pmatrix} p \\ q \end{pmatrix}\) and the value of \(t\).

**Markscheme**

\(\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}\), \(t = 3\)  
A1A1AI N3  
[3 marks]
Let \( f(x) = 4x - e^{x-2} - 3 \), for \( 0 \leq x \leq 5 \).

101a. Find the \( x \)-intercepts of the graph of \( f \).  

**Markscheme**

intercepts when \( f(x) = 0 \)  
\((0.827, 0) (4.78, 0) \) (accept \( x = 0.827, x = 4.78 \))  

**Examiners report**

[N/A]

101b. On the grid below, sketch the graph of \( f \).
Markscheme

Note: Award A1 for maximum point in circle, A1 for x-intercepts in circles, A1 for correct shape (y approximately greater than −3.14).

[3 marks]

Examiners report

[N/A]

Let \( h(x) = \frac{2x-1}{x+1}, \ x \neq -1 \).

102a. Find \( h^{-1}(x) \).

[4 marks]

Markscheme

\[ y = \frac{2x-1}{x+1} \]

interchanging \( x \) and \( y \) (seen anywhere) \( M1 \)

c.e.g. \( x = \frac{2y-1}{y+1} \)

correct working \( A1 \)

c.e.g. \( xy + x = 2y - 1 \)

collecting terms \( A1 \)

c.e.g. \( x + 1 = 2y - xy, \ x + 1 = y(2 - x) \)

\[ h^{-1}(x) = \frac{x+1}{2-x} \quad A1 \quad N2 \]

[4 marks]

Examiners report

[N/A]
102b. (i) Sketch the graph of $h$ for $-4 \leq x \leq 4$ and $-5 \leq y \leq 8$, including any asymptotes. [7 marks]

(ii) Write down the equations of the asymptotes.

(iii) Write down the $x$-intercept of the graph of $h$.

**Markscheme**

![Graph of $h$]

Note: Award $A1$ for approximately correct intercepts, $A1$ for correct shape, $A1$ for asymptotes, $A1$ for approximately correct domain and range.

(ii) $x = -1, y = 2$  $A1A1$  $N2$

(iii) $\frac{1}{2}$  $A1$  $N1$

[7 marks]

**Examiners report**

[N/A]

A rock falls off the top of a cliff. Let $h$ be its height above ground in metres, after $t$ seconds.

The table below gives values of $h$ and $t$.

<table>
<thead>
<tr>
<th>$t$ (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$ (metres)</td>
<td>105</td>
<td>98</td>
<td>84</td>
<td>60</td>
<td>26</td>
</tr>
</tbody>
</table>

103a. Jane thinks that the function $f(t) = -0.25t^3 - 2.32t^2 + 1.93t + 106$ is a suitable model for the data. Use Jane’s model to  [5 marks]

(i) write down the height of the cliff;

(ii) find the height of the rock after 4.5 seconds;

(iii) find after how many seconds the height of the rock is 30 m.

**Markscheme**

(i) 106 m  $A1$  $N1$

(ii) substitute $t = 4.5$  $MI$

$h = 44.9$ m  $A1$  $N2$

(iii) set up suitable equation  $MI$

e.g. $f(t) = 30$

$t = 4.91$  $A1$  $N1$

[5 marks]
103b. Kevin thinks that the function \( g(t) = -5.2t^2 + 9.5t + 100 \) is a better model for the data. Use Kevin’s model to find when the rock hits the ground.

**Markscheme**
recognizing that height is 0 \( A1 \)
set up suitable equation \( M1 \)
e.g. \( g(t) = 0 \)
\( t = 5.39 \) secs \( A1 \) \( N2 \)

[3 marks]

103c. (i) On graph paper, using a scale of 1 cm to 1 second, and 1 cm to 10 m, plot the data given in the table. [6 marks]
(ii) By comparing the graphs of \( f \) and \( g \) with the plotted data, explain which function is a better model for the height of the falling rock.

**Markscheme**

Note: Award \( A1 \) for correct scales on axes, \( A2 \) for 5 correct points, \( A1 \) for 3 or 4 correct points.

(ii) Jane’s function, with 2 valid reasons \( AIRIRI \) \( N3 \)
e.g. Jane’s passes very close to all the points, Kevin’s has the rock clearly going up initially – not possible if rock falls

Note: Although Jane’s also goes up initially, it only goes up very slightly, and so is the better model.

[6 marks]
The line $L$ passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

104a. Write down a vector equation for line $L$. [2 marks]

**Markscheme**

any correct equation in the form $\mathbf{r} = \mathbf{a} + t \mathbf{b}$ (accept any parameter for $t$)

where $\mathbf{a}$ is $\begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix}$, and $\mathbf{b}$ is a scalar multiple of $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$ A2 N2

e.g. $\mathbf{r} = \begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix} + t \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$, $\mathbf{r} = 5i - 4j + 10k + t(-8i + 4j - 10k)$

Note: Award A1 for the form $\mathbf{a} + t \mathbf{b}$, A1 for $L = \mathbf{a} + t \mathbf{b}$, A0 for $\mathbf{r} = \mathbf{b} + t \mathbf{a}$.

[2 marks]

Examiners report

In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "$L =$", rather than the mathematically correct "$\mathbf{r} =$" or "$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ =". $\mathbf{r}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ represent vectors, whereas $L$ is simply the name of the line. For part (b), very few candidates recognized that a general point on the x-axis will be given by the vector $\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$. Common errors included candidates setting their equation equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, or $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, or even just the number 0.

104b. The line $L$ intersects the x-axis at the point P. Find the x-coordinate of P. [6 marks]
**Markscheme**

recognizing that \( y = 0 \) or \( z = 0 \) at \( x \)-intercept (seen anywhere) \((R1)\)

attempt to set up equation for \( x \)-intercept (must suggest \( x \neq 0 \)) \((MI)\)

\[
e.g. \ L = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, \ 5 + 4t = x, \ r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
\]

one correct equation in one variable \((AI)\)

\[
e.g. -4 - 2t = 0, \ 10 + 5t = 0
\]

finding \( t = -2 \) \((AI)\)

correct working \((AI)\)

\[
e.g. x = 5 + (-2)(4)
\]

\( x = -3 \) (accept \((-3, 0, 0)\)) \((AI)\) \((N3)\)

\([6 \text{ marks}]\)

**Examiners report**

In part (a), the majority of candidates correctly recognized the equation that contains the position and direction vectors of a line. However, we saw a large number of candidates who continue to write their equations using "\( L = \)" rather than the mathematically correct "\( r = \)". \( r \) and \( y \) represent vectors, whereas \( L \) is simply the name of the line. For part (b), very few candidates recognized that a general point on the \( x \)-axis will be given by the vector \( \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \). Common errors included candidates setting their equation equal to \( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \), or \( \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \), or even just the number 0.

Let \( A \) and \( B \) be points such that \( \overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \) and \( \overrightarrow{OB} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} \).

105a.

Show that \( \overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \). \([1 \text{ mark}]\)
### Markscheme

correct approach  A1

e.g. \[ \overrightarrow{AO} + \overrightarrow{OB}, \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \]

\[ \overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \]  AG  N0

[1 mark]

### Examiners report

Part (a) was answered correctly by nearly every candidate.

105b. Let C and D be points such that ABCD is a rectangle.

Given that \( \overrightarrow{AD} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix} \), show that \( p = 3 \).

### Markscheme

recognizing \( \overrightarrow{AD} \) is perpendicular to \( \overrightarrow{AB} \) (may be seen in sketch)  (RI)
e.g. adjacent sides of rectangle are perpendicular
recognizing dot product must be zero  (RI)
e.g. \( \overrightarrow{AD} \cdot \overrightarrow{AB} = 0 \)
correct substitution  (AI)
e.g. \( (1 \times 4) + (-2 \times p) + (2 \times 1) = 4 - 2p + 2 = 0 \)
equation which clearly leads to \( p = 3 \)  A1
e.g. \( 6 - 2p = 0 , 2p = 6 \)

\( p = 3 \)  AG  N0

[4 marks]

### Examiners report

In part (b), the candidates who realized that the vectors must be perpendicular were successful using the scalar product to find \( p \). Incorrect approaches included using magnitudes, or creating vector equations of lines for the sides and setting them equal to each other. In addition, there were a good number of candidates who worked backwards, using the given value of 3 for \( p \) to find the coordinates of point D. Candidates who work backwards on a "show that" question will earn no marks.

105c. Let C and D be points such that ABCD is a rectangle.

Find the coordinates of point C.
**Markscheme**

**correct approach (seen anywhere including sketch) (AI)**

- \( \overrightarrow{O'C} = \overrightarrow{OB} + \overrightarrow{BC} \), \( \overrightarrow{OD} + \overrightarrow{DC} \)

**recognizing opposite sides are equal vectors (may be seen in sketch) (RI)**

- \( \overrightarrow{BC} = \overrightarrow{AD} \), \( \overrightarrow{DC} = \overrightarrow{AB} \)

coordinates of point C are \( (10, 3, 4) \) (accept \( (10, 3, 4) \)) \( A2 \) \( N4 \)

**Note:** Award \( A1 \) for two correct values.

[4 marks]

**Examiners report**

Part (c) was more difficult for candidates, and was left blank by some. Some candidates found \( \overrightarrow{AC} \) rather than \( \overrightarrow{OC} \), as required. Many candidates recognized that the opposite sides of the rectangle must be equal, but did not consider the directions of the vectors for those sides. There were also a good number of candidates who mislabelled the vertices of their rectangles, which led to them working with a rectangle ABDC, rather than ABCD.

105d. Let C and D be points such that ABCD is a **rectangle**. [5 marks]

Find the area of rectangle ABCD.

**Markscheme**

**attempt to find one side of the rectangle (MI)**

- \( e.g. \sqrt{1^2 + (-2)^2 + 2^2} \), \( 3 \); \( \sqrt{16 + 9 + 1} \), \( \sqrt{26} \)

**two correct magnitudes (AI)\( \)**

- \( e.g. \sqrt{26} \times \sqrt{9} \)

area = \( 3 \times \sqrt{26} \) (accept \( 3 \times \sqrt{26} \)) \( A1 \) \( N3 \)

[5 marks]

**Examiners report**

The majority of candidates who attempted part (d) were successful in multiplying the magnitudes of the sides. Unfortunately, there were some who set up their solutions correctly, then had arithmetic errors in their working.
The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and \( \overrightarrow{OA} = 4i \), \( \overrightarrow{OC} = 3j \), \( \overrightarrow{OD} = 2k \).

106a. (i) Find \( \overrightarrow{OB} \).

(ii) Find \( \overrightarrow{OF} \).

(iii) Show that \( \overrightarrow{AG} = -4i + 3j + 2k \).

 Marks scheme

(i) valid approach \( (M1) \)
e.g. \( \overrightarrow{OA} + \overrightarrow{OB} \)
\( \overrightarrow{OB} = 4i + 3j \) \( A1 \) \( N2 \)

(ii) valid approach \( (M1) \)
e.g. \( \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BF} ; \overrightarrow{OB} + \overrightarrow{BF} ; \overrightarrow{OC} + \overrightarrow{CG} + \overrightarrow{GF} \)
\( \overrightarrow{OF} = 4i + 3j + 2k \) \( A1 \) \( N2 \)

(iii) correct approach \( A1 \)
e.g. \( \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CG} ; \overrightarrow{AB} + \overrightarrow{BF} + \overrightarrow{FG} ; \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CG} \)
\( \overrightarrow{AG} = -4i + 3j + 2k \) \( AG \) \( N0 \)

[5 marks]

 Examiners report

Although a large proportion of candidates managed to answer this question, their biggest challenge was the use of a proper notation to represent the vectors and vector equations of lines.

In part (a), finding \( \overrightarrow{OB} \) and \( \overrightarrow{OF} \) was generally well done, although many lost the mark for (iii) due to poor working or not clearly showing the result.

106b. Write down a vector equation for

(i) the line OF;

(ii) the line AG.

[4 marks]
Markscheme

(i) any correct equation for (OF) in the form \( r = a + tb \quad A2 \quad N2 \)

where \( a \) is 0 or \( 4i + 3j + 2k \), and \( b \) is a scalar multiple of \( 4i + 3j + 2k \)

\[ \text{e.g. } r = t(4, 3, 2), \quad r = \left( 4t \right) \begin{pmatrix} 4t \end{pmatrix}, \quad r = 4i + 3j + 2k + t(4i + 3j + 2k) \]

(ii) any correct equation for (AG) in the form \( r = a + sb \quad A2 \quad N2 \)

where \( a \) is \( 4i \) or \( 3j \) and \( b \) is a scalar multiple of \(-4i + 3j + 2k\)

\[ \text{e.g. } r = (4, 0, 0) + s(-4, 3, 2), \quad r = \left( \begin{pmatrix} 4 - 4s \\ 3s \\ 2s \end{pmatrix} \right), \quad r = 3j + 2k + s(-4i + 3j + 2k) \]

[4 marks]

Examiners report

Part (b) was very poorly done. Not all the students recognized which correct position vectors they had to use to write the equations of the lines. It was seen that they frequently failed to present the equations in the required format, which prevented these candidates from achieving full marks. The notations generally seen were \( \overrightarrow{AG} = a + bt \), \( r = 4 + t(4, 3, 2) \) or \( L = a + bt \).

106c. Find the obtuse angle between the lines OF and AG. [7 marks]

Markscheme

choosing correct direction vectors, \( \overrightarrow{OF} \) and \( \overrightarrow{AG} \) \( (AI)(AI) \)

scalar product \( = -16 + 9 + 4 \quad (A1) \)

magnitudes \( \sqrt{4^2 + 3^2 + 2^2}, \sqrt{(-4)^2 + 3^2 + 2^2}, (\sqrt{29}, \sqrt{29}) \quad (AI)(AI) \)

substitution into formula \( M1 \)

\[ \text{e.g. } \cos \theta = \frac{-16 + 9 + 4}{(\sqrt{4^2 + 3^2 + 2^2})(\sqrt{(-4)^2 + 3^2 + 2^2})} = \left( \frac{1}{\sqrt{29}} \right) \]

\( 95.93777^\circ, 1.67443 \) radians \( \theta = 95.9^\circ \) or 1.67 \( A1 \quad N4 \)

[7 marks]

Examiners report

Most achieved the correct result in part (c) with many others gaining most of the marks as follow through from choosing incorrect vectors. Some students did not state which vectors had been used, another cause for losing marks. A few showed poor notation, including \( i, j \) and \( k \) in the working.

A line \( L_1 \) passes though points \( P(-1, 6, -1) \) and \( Q(0, 4, 1) \).

107a.

(i) Show that \( \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \). [3 marks]

(ii) Hence, write down an equation for \( L_1 \) in the form \( r = a + tb \).
**Markscheme**

(i) evidence of correct approach \( A1 \)

\( \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}, Q - P \)

\( \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad AG \quad N0 \)

(ii) any correct equation in the form \( r = a + tb \) \( A2 \quad N2 \)

where \( a \) is either \( \overrightarrow{OP} \) or \( \overrightarrow{OQ} \) and \( b \) is a scalar multiple of \( \overrightarrow{PQ} \)

\( e.g. \overrightarrow{Q} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, r = t \begin{pmatrix} 4 - 2t \\ 1 + 2t \end{pmatrix}, r = 4j + k + t(-2j + 2k) \)

[3 marks]

**Examiners report**

A pleasing number of candidates were successful on this straightforward vector and line question. Part (a) was generally well answered, although a few candidates still labelled their line \( L = \) or used a position vector for the direction vector. Follow-through marking allowed full recovery from the latter error.

A second line \( L_2 \) has equation \( r = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \).

107b. Find the cosine of the angle between \( \overrightarrow{PQ} \) and \( L_2 \). \[ 7 \text{ marks} \]

**Markscheme**

choosing a correct direction vector for \( L_2 \) \( A1 \)

\( e.g. \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \)

finding scalar products and magnitudes \( A1(A1)(A1) \)

scalar product = \( 1(3) - 2(0) + 2(-4) = -5 \)

magnitudes = \( \sqrt{3^2 + 0^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \)

substitution into formula \( M1 \)

\( e.g. \cos \theta = \frac{-5}{\sqrt{9} \sqrt{25}} \)

\( \cos \theta = -\frac{1}{3} \quad A2 \quad N5 \)

[7 marks]

**Examiners report**

Few candidates wrote down their direction vector in part (b) which led to lost follow-through marks, and a common error was finding an incorrect scalar product due to difficulty multiplying by zero.

107c. The lines \( L_1 \) and \( L_2 \) intersect at the point \( R \). Find the coordinates of \( R \). \[ 7 \text{ marks} \]
Markscheme

evidence of valid approach \((M1)\)
e.g. equating lines, \(L_1 = L_2\)

**EITHER**

one correct equation in one variable \((A2)\)
e.g. \(6 - 2t = 2\)

**OR**

two correct equations in two variables \((A1A1)\)
e.g. \(2t + 4s = 0\), \(t - 3s = 5\)

THEN

attempt to solve \((M1)\)

one correct parameter \((A1)\)
e.g. \(t = 2\), \(s = -1\)

correct substitution of either parameter \((A1)\)
e.g. \(r = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + (-1) \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, r = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + (+2) \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}\)

coordinates \(R(1, 2, 3)\) \((A1)\) \((N3)\)

[7 marks]

Examiners report

Part (c) was generally well understood with some candidates realizing that the equation in just one variable led to the correct parameter more quickly than solving a system of two equations to find both parameters. Some candidates gave the answer as \((s, t)\) instead of substituting those parameters, indicating a more rote understanding of the problem. Another common error was using the same parameter for both lines.

There were an alarming number of misreads of negative signs from the question or from the candidate working.

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The line \(L_1\) passes through the points \(P(2, 4, 8)\) and \(Q(4, 5, 4)\).

108a. (i) Find \(\overrightarrow{PQ}\) \([4\text{ marks}]\).

(ii) Hence write down a vector equation for \(L_1\) in the form \(r = a + sb\).
**Markscheme**

(i) evidence of approach \((M1)\)

\[ \overrightarrow{PQ} = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \] \(A1 \quad N2\)

(ii) any correct equation in the form \(r = a + sb\) (accept any parameter for \(s\))

where \(a\) is \(\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}\) or \(\begin{pmatrix} 4 \\ 5 \\ 4 \end{pmatrix}\), and \(b\) is a scalar multiple of \(\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \) \(A2 \quad N2\)

\[ e.g. \quad r = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \]

Note: Award \(A1\) for the form \(a + sb\), \(A1\) for \(L = a + sb\), \(A0\) for \(r = b + sa\).

[4 marks]

**Examiners report**

In part (a), nearly all the candidates correctly found the vector \(PQ\), and the majority went onto find the correct vector equation of the line. There are still many candidates who do not write this equation in the correct form, using "\(=\)", and these candidates were penalized one mark.

The line \(L_2\) is perpendicular to \(L_1\), and parallel to \(\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}\), where \(p \in \mathbb{Z}\).

108b. (i) Find the value of \(p\). \(\quad [7 \text{ marks}]\)

(ii) Given that \(L_2\) passes through \(R(10, 6, -40)\), write down a vector equation for \(L_2\).

**Markscheme**

(i) choosing correct direction vectors for \(L_1\) and \(L_2\) \((A1) \quad (A1)\)

\[ e.g. \quad \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix} \]

evidence of equating scalar product to 0 \((M1)\)

correct calculation of scalar product \(A1\)

\[ e.g. \quad 2 \times 3p + 1 \times 2p + (-4) \times 4 = 8p - 16 = 0 \]

\(p = 2 \quad A1 \quad N3\)

(ii) any correct expression in the form \(r = a + tb\) (accept any parameter for \(t\))

where \(a\) is \(\begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix}\), and \(b\) is a scalar multiple of \(\begin{pmatrix} 6 \\ 4 \end{pmatrix} \) \(A2 \quad N2\)

\[ e.g. \quad r = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + t \begin{pmatrix} 6 \\ 4 \end{pmatrix} \]

\[ = \begin{pmatrix} 10 + 6s \\ 6 + 4s \\ -40 + 4s \end{pmatrix} \]

\(r = 10i + 6j - 40k + s(6i + 4j + 4k) \)

Note: Award \(A1\) for the form \(a + tb\), \(A1\) for \(L = a + tb\) (unless they have been penalised for \(L = a + sb\) in part (a)), \(A0\) for \(r = b + ta\).

[7 marks]
Examiners report
In part (b), the majority of candidates knew to set the scalar product equal to zero for the perpendicular vectors, and were able to find the correct value of $p$.

108c. The lines $L_1$ and $L_2$ intersect at the point $A$. Find the $x$-coordinate of $A$. [7 marks]

**Markscheme**
appropriate approach \((MI)\)
e.g. \[
\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ -40 \end{pmatrix} + t \begin{pmatrix} 6 \\ 4 \end{pmatrix}
\]
any two correct equations with different parameters \((A1A1)\)
e.g. \[2 + 2s = 10 + 6t, 4 + s = 6 + 4t, 8 - 4s = -40 + 4t\]
attempt to solve simultaneous equations \((MI)\)
correct working \((AI)\)
e.g. \[-6 = -2 - 2t, 4 = 2t, -4 + 5s = 46, 5s = 50\]
one correct parameter \(s = 10, t = 2\) \((AI)\)
\(x = 22\) (accept \((22, 14, -32)\)) \((AI)\) \((N4)\)

[7 marks]

Examiners report
A good number of candidates used the correct method to find the intersection of the two lines, though some algebraic and arithmetic errors kept some from finding the correct final answer.

A line $L$ passes through $A(1, -1, 2)$ and is parallel to the line $r = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

109a. Write down a vector equation for $L$ in the form $r = a + tb$. [2 marks]

**Markscheme**
correct equation in the form $r = a + tb$ \((A2)\) \((N2)\)
\[
\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}
\]
[2 marks]

Examiners report
Many candidates answered this question well. Some continue to write the vector equation in (a) using "$L =$", which does not earn full marks.
109b. Find
(i) $\overrightarrow{OP}$;
(ii) $|\overrightarrow{OP}|$.

Markscheme
(i) attempt to substitute $t = 2$ into the equation (M1)
\[ e.g. \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \]
$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$  A1 N2

(ii) correct substitution into formula for magnitude  A1
\[ e.g. \sqrt{3^2 + 5^2 + (-2)^2}, \sqrt{3^2 + 5^2 + 2^2} \]
$|\overrightarrow{OP}| = 38$  A1 N1

Examiners report
Part (b) proved accessible for most, although small arithmetic errors were not uncommon. Some candidates substituted $t = 2$ into the original equation, and a few answered $\overrightarrow{OP} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix}$. A small but surprising number of candidates left this question blank, suggesting the topic was not given adequate attention in course preparation.

The following diagram shows the obtuse-angled triangle ABC such that $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}$.

110a. (i) Write down $\overrightarrow{BA}$.
(ii) Find $\overrightarrow{BC}$.
Markscheme

(i) \( \overrightarrow{BA} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} \)  
\( A1 \ N1 \)

(ii) evidence of combining vectors  \( (M1) \)
e.g. \( \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \), \( \overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{AB} \), \( \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix} + \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix} \)
\( \overrightarrow{BC} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \)  
\( A1 \ N2 \)

\[ 3 \text{ marks} \]

Examiners report

Many candidates answered (a) correctly, although some reversed the vectors when finding \( \overrightarrow{BC} \), while others miscopied the vectors from the question paper.

110 b. (i) Find \( \cos \angle ABC \).  
\( [7 \text{ marks}] \)

(ii) Hence, find \( \sin \angle ABC \).

Markscheme

(i) METHOD 1
finding \( \overrightarrow{BA} \cdot \overrightarrow{BC} \), \( |\overrightarrow{BA}| \), \( |\overrightarrow{BC}| \)
e.g. \( \overrightarrow{BA} \cdot \overrightarrow{BC} = 3 \times 1 + 0 \times 4 - 2 \times -2 \), \( |\overrightarrow{BA}| = \sqrt{3^2 + 4^2} \), \( |\overrightarrow{BC}| = 3 \)
substituting into formula for \( \cos \theta \)  \( M1 \)
e.g. \( \frac{3 \times 1 + 0 \times 4 - 2}{3 \sqrt{3^2 + 4^2} \cdot 5 \sqrt{3}} = \frac{5}{15} \)
\( \cos \angle ABC = -\frac{5}{15} \left( = -\frac{1}{3} \right) \)  
\( A1 \ N3 \)

METHOD 2
finding \( |\overrightarrow{AC}| \), \( |\overrightarrow{BA}| \), \( |\overrightarrow{BC}| \)  \( (AI)(AI)(AI) \)
e.g. \( |\overrightarrow{AC}| = \sqrt{2^2 + 2^2 + 6^2} \), \( |\overrightarrow{BA}| = \sqrt{3^2 + 4^2} \), \( |\overrightarrow{BC}| = 3 \)
substituting into cosine rule  \( M1 \)
e.g. \( \frac{9 \times 2 - 6}{2 \times 5 \times 3} = \frac{9}{30} \)
\( \cos \angle ABC = -\frac{9}{30} \left( = -\frac{1}{3} \right) \)  
\( A1 \ N3 \)

(ii) evidence of using Pythagoras  \( (M1) \)
e.g. right-angled triangle with values, \( \sin^2 x + \cos^2 x = 1 \)
\( \sin \angle ABC = \frac{\sqrt{8}}{3} \left( = \frac{2\sqrt{2}}{3} \right) \)  
\( A1 \ N2 \)

\[ 7 \text{ marks} \]

Examiners report

Students had no difficulty finding the scalar product and magnitudes of the vectors used in finding the cosine. However, few recognized that \( \overrightarrow{BA} \) is the vector to apply in the formula to find the cosine value. Most used \( \overrightarrow{AB} \) to obtain a positive cosine, which neglects that the angle is obtuse and thus has a negative cosine. Surprisingly few students could then take a value for cosine and use it to find a value for sine. Most left (bii) blank entirely.
110c. The point D is such that \( \overrightarrow{CD} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix} \), where \( p > 0 \).

(i) Given that \( |\overrightarrow{CD}| = \sqrt{50} \), show that \( p = 3 \).

(ii) Hence, show that \( \overrightarrow{CD} \) is perpendicular to \( \overrightarrow{BC} \).

**Markscheme**

(i) attempt to find an expression for \( |\overrightarrow{CD}| \) \( (M1) \)

e.g. \( \sqrt{(-4)^2 + 5^2 + p^2} \), \( |\overrightarrow{CD}|^2 = 4^2 + 5^2 + p^2 \)

correct equation \( A1 \)

e.g. \( \sqrt{(-4)^2 + 5^2 + p^2} = \sqrt{50} \), \( 4^2 + 5^2 + p^2 = 50 \)

\( p^2 = 9 \) \( A1 \)

\( p = 3 \) \( AG \) \( N0 \)

(ii) evidence of scalar product \( (M1) \)

e.g. \( \begin{pmatrix} -4 \\ 5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \overrightarrow{CD} \cdot \overrightarrow{BC} \)

correct substitution

e.g. \( -4 \times 1 + 5 \times 2 + 3 \times -2 = -4 + 10 - 6 \) \( A1 \)

\( \overrightarrow{CD} \cdot \overrightarrow{BC} = 0 \) \( A1 \)

\( \overrightarrow{CD} \) is perpendicular to \( \overrightarrow{BC} \) \( AG \) \( N0 \)

[6 marks]

**Examiners report**

Part (c) proved accessible for many candidates. Some created an expression for \( |\overrightarrow{CD}| \) and then substituted the given \( p = 3 \) to obtain \( \sqrt{50} \), which does not satisfy the "show that" instruction. Many students recognized that the scalar product must be zero for vectors to be perpendicular, and most provided the supporting calculations.

The following diagram shows quadrilateral ABCD, with \( \overrightarrow{AD} = \overrightarrow{BC} \), \( \overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \), and \( \overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \).

![Diagram](image)

111a. Find \( \overrightarrow{BC} \). [2 marks]
111b. Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

Markscheme

METHOD 1

$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ (AI)

correct approach $A1$

e.g. $\overrightarrow{AD} - \overrightarrow{AB} = \begin{pmatrix} 1 - 3 \\ 3 - 1 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG $N0$

METHOD 2

recognizing $\overrightarrow{CD} = -\overrightarrow{BA}$ (AI)

correct approach $A1$

e.g. $\overrightarrow{BC} + \overrightarrow{CD} = \begin{pmatrix} 1 - 3 \\ 3 - 1 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ AG $N0$

Examiners report

This question on two-dimensional vectors was generally very well done. A very small number of candidates had trouble with the "show that" in part (b) of the question.

111c. Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular.
Consider the vectors \( a = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \) and \( b = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \).

112. (a) Find

(i) \( 2a + b ; \)

(ii) \( |2a + b| . \)

Let \( 2a + b + c = 0 \), where \( 0 \) is the zero vector.

(b) Find \( c \).
**Markscheme**

(a)  
(i) \(2a = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \) \( (A1) \)  
correct expression for \(2a + b\) \( A1 \) \( N2 \)  
\(eg\) \( \begin{pmatrix} 5 \\ -2 \end{pmatrix} , (5, -2) , 5i - 2j \)

(ii) correct substitution into length formula \( (A1) \)  
\(eg\) \( \sqrt{5^2 + 2^2} \) \( , \sqrt{5^2 + -2^2} \)  
\(|2a + b| = \sqrt{29} \) \( A1 \) \( N2 \)  

[4 marks]

(b) valid approach \( (M1) \)  
\(eg\) \( c = -(2a + b) , 5 + x = 0 , -2 + y = 0 \)  
\(c = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \) \( A1 \) \( N2 \)  

[2 marks]

**Examiners report**

Most candidates comfortably applied algebraic techniques to find new vectors. However, a significant number of candidates answered part (b) as the absolute numerical value of the vector components, which suggests a misunderstanding of the modulus notation. Those who understood the notation easily made the calculation.

Consider points A(1, -2, -1), B(7, -4, 3) and C(1, -2, 3). The line \(L_1\) passes through C and is parallel to \(\overrightarrow{AB}\).

113a. Find \(\overrightarrow{AB}\).  

[2 marks]

**Markscheme**

valid approach \( (M1) \)  
\(eg\) \( \begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} , A - B , \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \)  
\(\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} \) \( A1 \) \( N2 \)  

[2 marks]

**Examiners report**

While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line.

113b. Hence, write down a vector equation for \(L_1\).  

[2 marks]
**Markscheme**

any correct equation in the form \( r = a + tb \) (accept any parameter for \( t \))

where \( a = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \) and \( b \) is a scalar multiple of \( \overrightarrow{AB} \)

\[ eg \quad r = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} , \ (x,y,z) = (1,-2,3) + t(3,-1,2) , \ r = \begin{pmatrix} 1 + 6t \\ -2 - 2t \\ 3 + 4t \end{pmatrix} \]

**Note:** Award \( A1 \) for \( a + tb \), \( A1 \) for \( L_1 = a + tb \), \( A0 \) for \( r = b + ta \).

[2 marks]

**Examiners report**

While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line. Most candidates wrote their equation as “\( \overrightarrow{L_1} = \)”, which misrepresents that the resulting equation must still be a vector.

A second line, \( L_2 \), is given by \( r = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix} \).

**Markscheme**

recognizing that scalar product \( = 0 \) (seen anywhere) \( R1 \)

correct calculation of scalar product \( \ (AI) \)

\[ eg \quad 6(3) - 2(-3) + 4p , 18 + 6 + 4p \]

correct working \( A1 \)

\[ eg \quad 24 + 4p = 0 , 4p = -24 \]

\( p = -6 \) \( AG \) \( N0 \)

[3 marks]

**Examiners report**

Those who recognized that vector perpendicularity means the scalar product is zero found little difficulty answering part (b). Occasionally a candidate would use the given \( p = 6 \) to show the scalar product is zero. However, working backward from the given answer earns no marks in a question that requires candidates to show that this value is achieved.

[7 marks]

113c. Given that \( L_1 \) is perpendicular to \( L_2 \), show that \( p = -6 \).

113d. The line \( L_1 \) intersects the line \( L_2 \) at point Q. Find the \( x \)-coordinate of Q.
**Markscheme**

setting lines equal  \((M1)\)

\[ \begin{align*}
\text{eg} & \quad L_1 = L_2, \quad \begin{pmatrix} 1 \\ -2 \\ 3 \\ 6 \\ 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 4 \\ 2 \\ 15 \\ -6 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \\
\text{any two correct equations with different parameters} & \quad \text{AIAI} \\
\text{eg} & \quad 1 + 6t = 1 + 3s, \quad -2 - 2t = 2 - 3s, \quad 3 + 4t = 15 - 6s \\
\text{attempt to solve their simultaneous equations} & \quad (M1) \\
\text{one correct parameter} & \quad \text{A1} \\
\text{eg} & \quad t = \frac{1}{2}, \quad s = \frac{5}{3} \\
\text{attempt to substitute parameter into vector equation} & \quad (M1) \\
\text{eg} & \quad \begin{pmatrix} 1 \\ -2 \\ 3 \\ 6 \\ 4 \\ \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}, \quad 1 + \frac{1}{2} \times 6 \\
x & = 4 \text{ (accept (4, -3, 5), ignore incorrect values for y and z)} \quad \text{A1 N3} \\
\end{align*} \]

[7 marks]

**Examiners report**

While many candidates knew to set the lines equal to find an intersection point, a surprising number could not carry the process to correct completion. Some could not solve a simultaneous pair of equations, and for those who did, some did not know what to do with the parameter value. Another common error was to set the vector equations equal using the same parameter, from which the candidates did not recognize a system to solve. Furthermore, it is interesting to note that while only one parameter value is needed to answer the question, most candidates find or attempt to find both, presumably out of habit in the algorithm.

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114.

Line \(L_1\) has equation \(r_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}\) and line \(L_2\) has equation \(r_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}\). Lines \(L_1\) and \(L_2\) intersect at point A. Find the coordinates of A.

**Markscheme**

appropriate approach  \((M1)\)

\[ \begin{align*}
\text{eg} & \quad \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}. \quad L_1 = L_2 \\
\text{any two correct equations} & \quad \text{AIAI} \\
\text{eg} & \quad 10 + 2s = 2 + 3t, \quad 6 - 5s = 1 + 5t, \quad -1 - 2s = -3 + 2t \\
\text{attempt to solve} & \quad (M1) \\
\text{eg substituting one equation into another} \\
\text{one correct parameter} & \quad \text{A1} \\
\text{eg} & \quad s = -1, \quad t = 2 \\
\text{correct substitution} & \quad \text{(A1)} \\
\text{eg} & \quad 2 + 3(2), \quad 1 + 5(2), \quad -3 + 2(2) \\
A & = (8, 11, 1) \text{ (accept column vector)} \quad \text{A1 N4} \\
\end{align*} \]

[7 marks]
Examiners report

Most students were able to set up one or more equations, but few chose to use their GDCs to solve the resulting system. Algebraic errors prevented many of these candidates from obtaining the final three marks. Some candidates stopped after finding the value of s and/or t.

The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, −1).

115a.
(i) Show that \( \overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \).

(ii) Find \( \overrightarrow{BD} \).

(iii) Show that \( \overrightarrow{AC} \) is perpendicular to \( \overrightarrow{BD} \).

Markscheme

(i) correct approach \( A1 \)

e.g. \( \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)

\( \overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \) \( AG \ N0 \)

(ii) appropriate approach \( M1 \)

e.g. \( \overrightarrow{D} - \overrightarrow{B} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} \), move 3 to the right and 6 down

\( \overrightarrow{BD} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \) \( A1 \ N2 \)

(iii) finding the scalar product \( A1 \)

e.g. \( 4(3) + 2(-6), 12 - 12 \)

valid reasoning \( R1 \)

e.g. \( 4(3) + 2(-6) = 0 \), scalar product is zero

\( \overrightarrow{AC} \) is perpendicular to \( \overrightarrow{BD} \) \( AG \ N0 \)

[5 marks]
Examiners report
The majority of candidates were successful on part (a), finding vectors between two points and using the scalar product to show two vectors to be perpendicular.

115b. The line (AC) has equation \( \mathbf{r} = \mathbf{u} + s \mathbf{v} \). [4 marks]

(i) Write down vector \( \mathbf{u} \) and vector \( \mathbf{v} \).

(ii) Find a vector equation for the line (BD).

Markscheme
(i) correct “position” vector for \( \mathbf{u} \); “direction” vector for \( \mathbf{v} \)  \( A1A1 \ N2 \)
e.g. \( \mathbf{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \); \( \mathbf{v} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \)
accept in equation e.g. \( \mathbf{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \mathbf{v} = t \begin{pmatrix} -4 \\ -2 \end{pmatrix} \)

(ii) any correct equation in the form \( \mathbf{r} = a + tb \), where \( \mathbf{b} = \mathbf{BD} \)
\( \mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -6 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \end{pmatrix} \)  \( A2 \ N2 \)

[4 marks]

Examiners report
Although a large number of candidates answered part (b) correctly, there were many who had trouble with the vector equation of a line. Most notably, there were those who confused the position vector with the direction vector, and those who wrote their equation in an incorrect form.

115c. The lines (AC) and (BD) intersect at the point \( P(3, k) \). [3 marks]
Show that \( k = 1 \).
MARKSCHEME

METHOD 1
substitute \((3, k)\) into equation for \((AC)\) or \((BD)\) \((M1)\)
e.g. \(3 = 1 + 4s\), \(3 = 1 + 3t\)
value of \(t\) or \(s\) \(A1\)
e.g. \(s = \frac{1}{2}, -\frac{1}{2}, t = \frac{2}{3}, -\frac{1}{3}\)
substituting \(A1\)
e.g. \(k = 0 + \frac{1}{2}(2)\)
\(k = 1\) \(AG\) \(N0\)

METHOD 2
setting up two equations \((M1)\)
e.g. \(1 + 4s = 4 + 3t\), \(2s = -1 - 6t\); setting vector equations of lines equal
value of \(t\) or \(s\) \(A1\)
e.g. \(s = \frac{1}{2}, -\frac{1}{2}, t = \frac{2}{3}, -\frac{1}{3}\)
substituting \(A1\)
e.g. \(r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 3 \\ -6 \end{pmatrix}\)
\(k = 1\) \(AG\) \(N0\)

[3 marks]

Examiners report
In part (c), most candidates seemed to know what was required, though there were many who made algebraic errors when solving for the parameters. A few candidates worked backward, using \(k = 1\), which is not allowed on a “show that” question.

115d. The lines \((AC)\) and \((BD)\) intersect at the point \(P(3, k)\) .

Hence find the area of triangle \(ACD\).

MARKSCHEME

\[\overrightarrow{PD} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}\] \((A1)\)
\[|\overrightarrow{PD}| = \sqrt{2^2 + (-2)^2} = \sqrt{4 + 4} \] \((A1)\)
\[|\overrightarrow{AC}| = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} \] \((A1)\)
area \(= \frac{1}{2} \times |\overrightarrow{AC}| \times |\overrightarrow{PD}| = \frac{1}{2} \times \sqrt{20} \times \sqrt{5} \) \(M1\)
\(= 5\) \(A1\) \(N4\)

[5 marks]

Examiners report
In part (d), candidates attempted many different geometric and vector methods to find the area of the triangle. As the question said "hence", it was required that candidates should use answers from their previous working - i.e. \(AC \perp BD\) and \(P(3, 1)\). Some geometric approaches, while leading to the correct answer, did not use "hence" or lacked the required justification.
The line $L_1$ is represented by the vector equation $\mathbf{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line $L_2$ is parallel to $L_1$ and passes through the point $B(-8, -5, 25)$.

**116a.** Write down a vector equation for $L_2$ in the form $\mathbf{r} = a + tb$.

**Markscheme**

any correct equation in the form $\mathbf{r} = a + tb$ (accept any parameter) \( A2 \quad N2 \)

e.g. $\mathbf{r} = \begin{pmatrix} -8 \\ -5 \\ 25 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$

**Note:** Award \( A1 \) for $a + tb$, \( A1 \) for $L = a + tb$, \( A0 \) for $r = b + ta$.

**2 marks**

**Examiners report**

Many candidates gave a correct vector equation for the line.

**116b.** A third line $L_3$ is perpendicular to $L_1$ and is represented by $\mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

Show that $k = -2$.

**Markscheme**

recognizing scalar product must be zero (seen anywhere) \( R1 \)
e.g. $\mathbf{a} \cdot \mathbf{b} = 0$

evidence of choosing direction vectors $\begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$, $\begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$ \( (A1)(A1) \)
correct calculation of scalar product \( (A1) \)
e.g. $2(-7) + 1(-2) - 8k$
simplification that clearly leads to solution \( A1 \)
e.g. $-16 - 8k$, $-16 - 8k = 0$
$k = -2 \quad AG \quad N0$

**5 marks**

**Examiners report**

A common error was to misplace the initial position and direction vectors. Those who set the scalar product of the direction vectors to zero typically solved for $k$ successfully. Those who substituted $k = -2$ earned fewer marks for working backwards in a "show that" question.

**116c.** The lines $L_1$ and $L_3$ intersect at the point $A$.

Find the coordinates of $A$. 

Find the coordinates of $A$. 

Find the coordinates of $A$. 

**Markscheme**

evidence of equating vectors \((M1)\)

e.g. \(L_1 = L_3 \cdot \left( \begin{array}{c} -3 \\ -1 \\ -25 \end{array} \right) + p \left( \begin{array}{c} 2 \\ 1 \\ -8 \end{array} \right) = \left( \begin{array}{c} 5 \\ 0 \\ 3 \end{array} \right) + q \left( \begin{array}{c} -7 \\ -2 \\ -2 \end{array} \right)\)

any two correct equations \(A1\)

e.g. \(-3 + 2p = 5 - 7q, -1 + p = -2q, -25 - 8p = 3 - 2q\)

attempting to solve equations \((M1)\)

finding one correct parameter \((p = -3, q = 2)\) \(A1\)

the coordinates of \(A\) are \((-9, -4, -1)\) \(A1\) \(N3\)

[6 marks]

**Examiners report**

Many went on to find the coordinates of point \(A\), however some used the same letter, say \(p\), for each parameter and thus could not solve the system.

**Markscheme**

(i) evidence of appropriate approach \((M1)\)

e.g. \(\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \), \(\overrightarrow{AB} = \left( \begin{array}{c} -8 \\ -5 \\ 25 \end{array} \right) - \left( \begin{array}{c} -9 \\ -4 \\ -1 \end{array} \right)\)

\(\overrightarrow{AB} = \left( \begin{array}{c} 1 \\ -1 \\ 26 \end{array} \right)\) \(A1\) \(N2\)

(ii) finding \(\overrightarrow{AC} = \left( \begin{array}{c} 7 \\ 2 \\ 2 \end{array} \right)\) \(A1\)

evidence of finding magnitude \((M1)\)

e.g. \(|\overrightarrow{AC}| = \sqrt{7^2 + 2^2 + 2^2}\)

\(|\overrightarrow{AC}| = \sqrt{57}\) \(A1\) \(N3\)

[5 marks]

**Examiners report**

Part (d) proved challenging as many candidates did not consider that \(\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\). Rather, many attempted to find the coordinates of point \(C\), which became a more arduous and error-prone task.
Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{\text{AC}} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

117a. Find $\overrightarrow{\text{BC}}$.

**Markscheme**

 evidence of appropriate approach  \((M1)\)  

 e.g. \(\overrightarrow{\text{BC}} = \overrightarrow{\text{BA}} + \overrightarrow{\text{AC}}\), \(\begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}\)  

\(\overrightarrow{\text{BC}} = \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}\)  \(A1\) \(N2\)

[2 marks]

**Examiners report**

Part (a) was generally done well with candidates employing different correct methods to find the vector $\overrightarrow{\text{BC}}$. Some candidates subtracted the given vectors in the wrong order and others simply added them. Calculation errors were seen with some frequency.

117b. Find a unit vector in the direction of $\overrightarrow{\text{AB}}$.

**Markscheme**

 attempt to find the length of $\overrightarrow{\text{AB}}$  \((M1)\)  

\(\overrightarrow{\text{AB}} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}\) \(l = \sqrt{6^2 + (-2)^2 + 3^2} = \sqrt{49} = 7\)  \(A1\)

unit vector is \(\frac{\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}}{7}\)  \(A1\) \(N2\)

[3 marks]

**Examiners report**

Many candidates did not appear to know how to find a unit vector in part (b). Some tried to write down the vector equation of a line, indicating no familiarity with the concept of unit vectors while others gave the vector \((1, 1, 1)\) or wrote the same vector $\overrightarrow{\text{AB}}$ as a linear combination of \(i\), \(j\) and \(k\). A number of candidates correctly found the magnitude but did not continue on to write the unit vector.

117c. Show that $\overrightarrow{\text{AB}}$ is perpendicular to $\overrightarrow{\text{AC}}$.

**Markscheme**

 recognizing that the dot product or \(\cos \theta\) being 0 implies perpendicular  \((M1)\)  

 correct substitution in a scalar product formula  \(A1\)  

 e.g. \((6) \times (-2) + (-2) \times (-3) + (3) \times (2) = \cos \theta = \frac{12 + 6 + 6}{7 \times \sqrt{17}}\)  

 correct calculation  \(A1\)  

 e.g. $\overrightarrow{\text{AB}} \cdot \overrightarrow{\text{AC}} = 0\), $\cos \theta = 0$  

 therefore, they are perpendicular  \(AG\) \(N0\)

[3 marks]
Examiners report
Candidates were generally successful in showing that the vectors in part (c) were perpendicular. Many used the efficient approach of showing that the scalar product equaled zero, while others worked a little harder than necessary and used the cosine rule to find the angle between the two vectors.

118a. Let \( u = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \) and \( w = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix} \). Given that \( u \) is perpendicular to \( w \), find the value of \( p \).

**Markscheme**
evidence of equating scalar product to 0 \( (M1) \)
\[
2 \times 3 + 3 \times ( -1) + ( -1) \times p = 0 \] \( (6 - 3 - p = 0, 3 - p = 0) \) \( A1 \)
p = 3 \( A1 \) \( N2 \)

[3 marks]

Examiners report
Most candidates knew to set the scalar product equal to zero.

118b. Let \( \mathbf{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix} \). Given that \( |\mathbf{v}| = \sqrt{42} \), find the possible values of \( q \).

**Markscheme**
evidence of substituting into magnitude formula \( (M1) \)
e.g. \( \sqrt{1 + q^2 + 25} \), \( 1 + q^2 + 25 \)
setting up a correct equation \( A1 \)
e.g. \( \sqrt{1 + q^2 + 25} = \sqrt{42} \), \( 1 + q^2 + 25 = 42 \), \( q^2 = 16 \)
q = \pm 4 \( A1 \) \( N2 \)

[3 marks]

Examiners report
Most candidates knew to set the scalar product equal to zero. A pleasing number found both answers for \( q \), although some often neglected to provide both solutions.

119a. Show that \( \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \).

**Markscheme**
evidence of correct approach \( A1 \)
e.g. \( \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} \), \( \left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right) - \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right) \)
\( \overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \) \( AG \) \( N0 \)

[1 mark]
Examiners report
Most candidates answered part (a) easily.

119b. The line \(L_1\) may be represented by

\[
\mathbf{r} = \left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right) + s \left( \begin{array}{c} 1 \\ -2 \\ 3 \end{array} \right) \] 

(i) What information does the vector \(\left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right)\) give about \(L_1\)?
(ii) Write down another vector representation for \(L_1\) using \(\left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right)\).

Markscheme
(i) correct description \(R1\) \(N1\)
\(\text{e.g. reference to \(\left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right)\) being the position vector of a point on the line, a vector to the line, a point on the line.}
(ii) any correct expression in the form \(\mathbf{r} = \mathbf{a} + t \mathbf{b}\) \(A2\) \(N2\)
where \(\mathbf{a}\) is \(\left( \begin{array}{c} 3 \\ -3 \\ 8 \end{array} \right)\), and \(\mathbf{b}\) is a scalar multiple of \(\left( \begin{array}{c} 1 \\ -2 \\ 3 \end{array} \right)\).
\(\text{e.g. } \left( \begin{array}{c} 3 + 2s \\ -3 - 4s \\ 8 + 6s \end{array} \right)\) \(\text{or } \left( \begin{array}{c} 3 + 2s \\ -3 - 4s \\ 8 + 6s \end{array} \right)\).

Examiners report
For part (b), a number of candidates stated that the vector was a "starting point," which misses the idea that it is a position vector to some point on the line.

119c. The point \(T(-1, 5, p)\) lies on \(L_1\).
Find the value of \(p\).

Markscheme
one correct equation \(A1\)
\(\text{e.g. } 3 + s = -1, -3 - 2s = 5\)
\(s = -4\) \(A1\)
\(p = -4\) \(A1\) \(N2\)

Examiners report
Parts (c) and (d) proved accessible to many.

119d. The point \(T\) also lies on \(L_2\) with equation

\[
\mathbf{r} = \left( \begin{array}{c} -3 \\ 9 \end{array} \right) + t \left( \begin{array}{c} 1 \\ -2 \end{array} \right) \]

Show that \(q = -3\).
Markscheme

one correct equation \( A1 \)

e.g. \(-3 + t = -1, 9 - 2t = 5\)

\( t = 2 \) \( A1 \)

substituting \( t = 2 \)

e.g. \( 2 + 2q = -4, 2q = -6 \) \( A1 \)

\( q = -3 \) \( AG \) \( N0 \)

[3 marks]

Examiners report

Parts (c) and (d) proved accessible to many.

119e. Let \( \theta \) be the obtuse angle between \( L_1 \) and \( L_2 \). Calculate the size of \( \theta \). [7 marks]

Markscheme

choosing correct direction vectors \( \left( \begin{array}{c} 1 \\ -2 \\ 3 \end{array} \right) \) and \( \left( \begin{array}{c} 1 \\ -2 \\ -3 \end{array} \right) \) \( (A1)(A1) \)

finding correct scalar product and magnitudes \( (A1)(A1)(A1) \)

scalar product \( (1)(1) + (-2)(-2) + (-3)(3) = -4 \)

magnitudes \( \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14} \)

evidence of substituting into scalar product \( M1 \)

e.g. \( \cos \theta = \frac{-4}{\sqrt{14} \times \sqrt{14}} \)

\( \theta = 1.86 \) radians \( \text{ (or } 107^\circ \text{ )} \) \( A1 \) \( N4 \)

[7 marks]

Examiners report

For part (e), a surprising number of candidates chose incorrect vectors. Few candidates seemed to have a good conceptual understanding of the vector equation of a line.

The vertices of the triangle PQR are defined by the position vectors

\( \overrightarrow{OP} = \left( \begin{array}{c} 4 \\ -3 \\ 1 \end{array} \right) \), \( \overrightarrow{OQ} = \left( \begin{array}{c} 3 \\ -1 \\ 2 \end{array} \right) \) and \( \overrightarrow{OR} = \left( \begin{array}{c} 6 \\ -1 \\ 5 \end{array} \right) \).

120a. Find [3 marks]

(i) \( \overrightarrow{PQ} \);

(ii) \( \overrightarrow{PR} \).
Markscheme
(i) evidence of approach \((M1)\)

\[ \overrightarrow{\text{PQ}} = \overrightarrow{\text{PO}} + \overrightarrow{\text{OQ}}, \quad \text{e.g.} \]

\[ \overrightarrow{\text{PQ}} = \left( \begin{array}{c} -1 \\ 2 \\ 1 \end{array} \right) \]

\((A1)\) \(N2\)

(ii) \[ \overrightarrow{\text{PR}} = \left( \begin{array}{c} 2 \\ 2 \\ 4 \end{array} \right) \]

\((A1)\) \(N1\)

[3 marks]

Examiners report
Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.

120b. Show that \(\cos \overrightarrow{\text{R}} \; \overrightarrow{\text{P}}\) = \(\frac{1}{2}\).

[7 marks]

Markscheme
METHOD 1

choosing correct vectors \(\overrightarrow{\text{PQ}}\) and \(\overrightarrow{\text{PR}}\) \((A1)(A1)\)

finding \(\overrightarrow{\text{PQ}} \; \bullet \; \overrightarrow{\text{PR}}, \; \left| \overrightarrow{\text{PQ}} \right|, \; \left| \overrightarrow{\text{PR}} \right|\)

\(\overrightarrow{\text{PQ}} \; \bullet \; \overrightarrow{\text{PR}} = -2 + 4 + 4(=6)\)

\(\left| \overrightarrow{\text{PQ}} \right| = \sqrt{(-1)^2 + 2^2 + 1^2} \quad (= \sqrt{6})\)

\(\left| \overrightarrow{\text{PR}} \right| = \sqrt{2^2 + 2^2 + 4^2} \quad (= \sqrt{24})\)

substituting into formula for angle between two vectors \(M1\)

e.g. \(\cos \overrightarrow{\text{R}} \; \overrightarrow{\text{P}} = \frac{\overrightarrow{\text{PQ}} \; \bullet \; \overrightarrow{\text{PR}}}{\left| \overrightarrow{\text{PQ}} \right| \times \left| \overrightarrow{\text{PR}} \right|}\)

\(= \frac{6}{\sqrt{6} \times \sqrt{24}}\), \(\frac{6}{\sqrt{144}}, \frac{6}{12}\)

\(\cos \overrightarrow{\text{R}} \; \overrightarrow{\text{P}} = \frac{1}{2}\) \((AG)\) \(N0\)

METHOD 2

evidence of choosing cosine rule (seen anywhere) \((M1)\)

\(\overrightarrow{\text{QR}} = \left( \begin{array}{c} 3 \\ 0 \\ 3 \end{array} \right)\)

\(\left| \overrightarrow{\text{QR}} \right| = \sqrt{18}, \; \left| \overrightarrow{\text{PQ}} \right| = \sqrt{6}\) \(\text{and} \; \left| \overrightarrow{\text{PR}} \right| = \sqrt{24}\) \((A1)(A1)(A1)\)

\(\cos \overrightarrow{\text{R}} \; \overrightarrow{\text{P}} = \frac{\sqrt{6}^2 + \sqrt{24}^2 - \sqrt{18}^2}{2 \sqrt{6} \times \sqrt{24}}\)

\(= \frac{6 + 24 - 18}{24}\)

\(= \frac{12}{24}\) \((A1)\)

\(\cos \overrightarrow{\text{R}} \; \overrightarrow{\text{P}} = \frac{1}{2}\) \((AG)\) \(N0\)

[7 marks]

Examiners report
Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although PQR is indeed right-angled, in a “show that” question this attribute must be directly established.

121. Let \(\overrightarrow{v} = \left( \begin{array}{c} 2 \\ -3 \\ 6 \end{array} \right)\) and \(\overrightarrow{w} = \left( \begin{array}{c} k \\ -2 \\ 4 \end{array} \right)\), for \(k > 0\). The angle between \(v\) and \(w\) is \(\frac{\pi}{3}\).

Find the value of \(k\).
Markscheme

correct substitutions for \(\{v\} \bullet \{w\}\); \(\left| \{v\} \right|\); \(\left| \{w\} \right|\) (A1)(A1)(A1)

e.g. \(2k + ( - 3) \times ( - 2) + 6 \times 4, 2k + 30; \sqrt {2^2 + (-3)^2 + 6^2}, \sqrt {49}; \sqrt {k^2 + (-2)^2 + 4^2}, \sqrt {k^2 + 20}\)

evidence of substituting into the formula for scalar product \((M1)\)

e.g. \(\frac{2k + 30}{7 \times \sqrt {k^2 + 20}}\)

correct substitution \(A1\)

e.g. \(\cos \frac{\pi}{3} = \frac{2k + 30}{7 \times \sqrt {k^2 + 20}}\)

\[7\text{ marks}\]

Examiners report

For the most part, this question was well done and candidates had little difficulty finding the scalar product, the appropriate magnitudes and then correctly substituting into the formula for the angle between vectors. However, few candidates were able to solve the resulting equation using their GDCs to obtain the correct answer. Problems arose when candidates attempted to solve the resulting equation analytically.

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122a.

\[4\text{ marks}\]

(i) Write down the coordinates of A.

(ii) Find the speed of the airplane in \(\text{m/s}\).

Markscheme

(i) \((3, -4, 0)\) \((A1)\)

(ii) choosing velocity vector \(\left( \begin{array}{c} -2 \\ 3 \\ 1 \end{array} \right)\) \((M1)\)

finding magnitude of velocity vector \((A1)\)

e.g. \(\sqrt {(-2)^2 + 3^2 + 1^2}, \sqrt {4 + 9 + 1}\)

speed = 3.74 \(\left( \sqrt{14} \right)\) \(A1\)

\[4\text{ marks}\]

Examiners report

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

122b.

\[5\text{ marks}\]

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

Markscheme

(i) \((3, -4, 0)\) \(A1\)

(ii) \(\left( \begin{array}{c} -2 \\ 3 \\ 1 \end{array} \right)\) \((M1)\)

finding magnitude of velocity vector \((A1)\)

e.g. \(\sqrt {(3^2) + (1^2)}, \sqrt {10}\)

speed = 3.74 \(\left( \sqrt{14} \right)\) \(A1\)

\[5\text{ marks}\]
**Markscheme**

(i) substituting \( p = 7 \) \( (M1) \)

\[
\text{B} = (-11, 17, 7) \quad A1 \quad N2
\]

(ii) **METHOD 1**

appropriate method to find \( \overrightarrow{AB} \) or \( \overrightarrow{BA} \) \( (M1) \)

e.g. \( \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \), \( \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} \)

\[
\overrightarrow{AB} = \left( \begin{array}{c} -14 \\ 21 \\ 7 \end{array} \right) \quad or \quad \overrightarrow{BA} = \left( \begin{array}{c} 14 \\ -21 \\ -7 \end{array} \right) \quad A1 \quad N2
\]

\[ \text{distance} = 26.2 \ \Left( 7\sqrt{14} \) \right) \quad A1 \quad N3

**METHOD 2**

evidence of applying distance is speed \( \times \) time \( (M2) \)

e.g. \( 3.74 \times 7 \)

\[ \text{distance} = 26.2 \ \Left( 7\sqrt{14} \) \right) \quad A1 \quad N3

**METHOD 3**

try to find \( AB \), \( AB \) \( (M1) \)

e.g. \( (3 - (-11))^2 + (-4 - 17)^2 + (0 - 7)^2 \), \( \sqrt{(3 - (-11))^2 + (-4 - 17)^2 + (0 - 7)^2} \)

\[ AB = 686, \quad AB = 7\sqrt{14} \quad A1 \quad N3 \]

[5 marks]

**Examiners report**

Many candidates demonstrated a good understanding of the vector equation of a line and its application to a kinematics problem by correctly answering the first two parts of this question.

Some knew that speed and distance were magnitudes of vectors but chose the wrong vectors to calculate magnitudes.

122c. Airplane 2 passes through a point C. Its position \( q \) seconds after it passes through C is given by \[
\left( \begin{array}{c} x \\ y \\ z \end{array} \right) = \left( \begin{array}{c} 2 \\ -5 \\ 8 \end{array} \right) + q\left( \begin{array}{c} -1 \\ 2 \\ a \end{array} \right), \ a \in \mathbb{R}.
\]

The angle between the flight paths of Airplane 1 and Airplane 2 is \( 40^\circ \). Find the two values of \( a \).

**Markscheme**

correct direction vectors \( \overrightarrow{AB} = \left( \begin{array}{c} -2 \\ 3 \\ 1 \end{array} \right) \) \( (A1) \)

\( \overrightarrow{CA} = \left( \begin{array}{c} a^2 + 5 \\ a \end{array} \right) \), \( \overrightarrow{CA} = a \ \sqrt{(a^2 + 5)} \)

\[ a = 3.21, \quad a = -0.990 \quad A1A1 \quad N3 \]

[7 marks]

**Examiners report**

Very few candidates were able to get the two correct answers in (c) even if they set up the equation correctly. Much contorted algebra was seen and little evidence of using the GDC to solve the equation. Many made simple algebraic errors by combining unlike terms in working with the scalar product (often writing \( 8a \) rather than \( 8 + a \)) or the magnitude (often writing \( 5(a^2) \) rather than \( 5 + a^2 \)).
Line $L_1$ passes through points $A(1, -1, 4)$ and $B(2, -2, 5)$.

123a. Find $\overrightarrow{AB}$.

**Markscheme**

appropriate approach $(M1)$

e.g. $\overrightarrow{AO} + \overrightarrow{OB}$, $B - A$

$\overrightarrow{AB} = \left( \begin{array}{c} 1 \\ -1 \\ 1 \end{array} \right)$  \hspace{1cm} A1 N2

[2 marks]

**Examiners report**

Finding $\overrightarrow{AB}$ was generally well done, although some candidates reversed the subtraction.

123b. Find an equation for $L_1$ in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$.

**Markscheme**

any correct equation in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$  \hspace{1cm} A2 N2

where $\mathbf{b}$ is a scalar multiple of $\overrightarrow{AB}$, e.g.

$\mathbf{r} = \left( \begin{array}{c} 1 \\ -1 \\ 4 \end{array} \right) + t\left( \begin{array}{c} 1 \\ -1 \\ 1 \end{array} \right)$, $\mathbf{r} = \left( \begin{array}{c} 2 + t \\ -2 - t \\ 5 + t \end{array} \right)$, $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$

[2 marks]

**Examiners report**

In part (b) not all the candidates recognized that $\overrightarrow{AB}$ was the direction vector of the line, as some used the position vector of point B as the direction vector.

123c. Find the angle between $L_1$ and $L_2$.

**Markscheme**

any correct approach $(A1)$

any correct equation for $L_2$ in the form $\mathbf{r} = \mathbf{a} + s\mathbf{b}$, e.g.

$\mathbf{r} = \left( \begin{array}{c} 2 \\ 4 \\ 7 \end{array} \right) + s\left( \begin{array}{c} 2 \\ 1 \\ 3 \end{array} \right)$

$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$

[7 marks]
Examiners report
Many candidates successfully used scalar product and magnitudes in part (c), although a large number did choose vectors other than the direction vectors and many did not state clearly which vectors they were using.

123d. The lines $[L_1]$ and $[L_2]$ intersect at point C. Find the coordinates of C. \[6 \text{ marks}\]

Markscheme

METHOD 1 (from $\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach (M1)

e.g. $\mathbf{p} = \mathbf{r}$, $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$

two correct equations (A1A1)

e.g. $1 + t = 2 + 2s$, $-1 - t = 4 + s$, $4 + t = 7 + 3s$

attempt to solve (M1)

one correct parameter (A1)

e.g. $t = -3$, $s = -2$

C is $(-2, 2, 1)$ (A1 N3)

METHOD 2 (from $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach (M1)

e.g. $\mathbf{p} = \mathbf{r}$, $\begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$

two correct equations (A1A1)

e.g. $2 + t = 2 + 2s$, $-2 - t = 4 + s$, $5 + t = 7 + 3s$

attempt to solve (M1)

one correct parameter (A1)

e.g. $t = -4$, $s = -2$

C is $(-2, 2, 1)$ (A1 N3)

[6 marks]

Examiners report
Candidates who were comfortable on the first three parts often had little difficulty with the final part. While the resulting systems were easily solved algebraically, a surprising number of candidates did not check their solutions either manually or with technology. An occasionally seen error in the final part was using a midpoint to find C. Some candidates found the point of intersection in part (c) rather than in part (d), indicating a familiarity with the type of question but a lack of understanding of the concepts involved.

A particle is moving with a constant velocity along line $L$. Its initial position is $A(6, -2, 10)$. After one second the particle has moved to $B(9, -6, 15)$.

124a. (i) Find the velocity vector, $\overrightarrow{AB}$. \[4 \text{ marks}\]

(ii) Find the speed of the particle.
**Markscheme**

(i) evidence of approach \( (M1) \)

e.g. \( \overrightarrow{AO} + \overrightarrow{OB} \), \( B - A \), \( \begin{array}{*{20}{c}} {9 - 6}\hfill \{ - 6 + 2\hfill \{ 15 - 10\end{array} \overrightarrow{AB} = \begin{array}{*{20}{c}} {3}\hfill \{ - 4\hfill \{ 5\end{array} \} \right) \) (accept \( \{ 3, - 4, 5\right) \) \( A1 \quad N2 \)

(ii) evidence of finding the magnitude of the velocity vector \( (M1) \)

e.g. \( \text{speed} = \sqrt{{3^2} + {4^2} + {5^2}} \)
\( \text{speed} = \sqrt{50} \) \( \right) \right) \) \( A1 \quad N1 \)

\([4 \text{ marks}]

**Examiners report**

This question was quite well done. Marks were lost when candidates found the vector \( \overrightarrow{BA} \) instead of \( \overrightarrow{AB} \) in part (a) and for not writing their vector equation as an equation.

124b. Write down an equation of the line \( L \). \([2 \text{ marks}]\)

**Markscheme**

correct equation (accept Cartesian and parametric forms) \( A2 \quad N2 \)

e.g. \( \mathbf{r} = \left( \begin{array}{*{20}{c}} {6}\hfill \{ - 2\hfill \{ 10\end{array} \right) + t\left( \begin{array}{*{20}{c}} {3}\hfill \{ - 4\hfill \{ 5\end{array} \right) \right) \right) \) \( A1 \quad N1 \)

\([2 \text{ marks}]

**Examiners report**

In part (b), a few candidates switched the position and velocity vectors or used the vectors \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \) to incorrectly write the vector equation.

The diagram shows a parallelogram ABCD.

![Diagram not to scale](image)

The coordinates of A, B and D are A(1, 2, 3), B(6, 4, 4) and D(2, 5, 5).

125a. (i) Show that \( \overrightarrow{AB} = \left( \begin{array}{*{20}{c}} {5}\hfill \{ 2\hfill \{ 1\end{array} \right) \right) \) \( A2 \quad N2 \)

(ii) Find \( \overrightarrow{AD} \).

(iii) Hence show that \( \overrightarrow{AC} = \left( \begin{array}{*{20}{c}} {6}\hfill \{ 5\hfill \{ 3\end{array} \right) \right) \) \( A2 \quad N2 \)

\([5 \text{ marks}]\)
Markscheme

(i) evidence of approach \( M1 \)

e.g. \( \overrightarrow{\text{BA}} \) or \( \overrightarrow{\text{OA}} + \overrightarrow{\text{OB}} \) or \( \begin{pmatrix} 6 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \) \( \overrightarrow{\text{AB}} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \)

(ii) evidence of approach \( (M1) \)

e.g. \( \overrightarrow{\text{DA}} \) or \( \overrightarrow{\text{OA}} + \overrightarrow{\text{OD}} \) or \( \begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \) \( \overrightarrow{\text{AD}} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \)

(iii) evidence of approach \( (M1) \)

e.g. \( \overrightarrow{\text{AC}} = \overrightarrow{\text{AB}} + \overrightarrow{\text{AD}} \) correct substitution \( A1 \)

e.g. \( \overrightarrow{\text{AC}} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \) \( \overrightarrow{\text{AC}} = \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} \)

[5 marks]

Examiners report

Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

125b. Find the coordinates of point C. \[3 \text{ marks}\]

Markscheme

evidence of combining vectors (there are at least 5 ways) \( (M1) \)

e.g. \( \overrightarrow{\text{OC}} = \overrightarrow{\text{OA}} + \overrightarrow{\text{AC}} \) or \( \overrightarrow{\text{OC}} = \overrightarrow{\text{OB}} + \overrightarrow{\text{AD}} \) or \( \overrightarrow{\text{AB}} = \overrightarrow{\text{OC}} - \overrightarrow{\text{OD}} \)

correct substitution \( A1 \)

e.g. coordinates of C are \( 7,7,6 \) \( A1 \)

[3 marks]

Examiners report

Candidates performed very well in this question, showing a strong ability to work with the algebra and geometry of vectors.

125c. (i) Find \( \overrightarrow{\text{AB}} \bullet \overrightarrow{\text{AD}} \). \[7 \text{ marks}\]

(ii) Hence find angle A.

Markscheme

(i) \( \overrightarrow{\text{AB}} \bullet \overrightarrow{\text{AD}} \) or \( \begin{vmatrix} 5 \\ 2 \\ 1 \end{vmatrix} \begin{vmatrix} 6 \\ 5 \\ 3 \end{vmatrix} \)

(ii) Hence find angle A.
Markscheme

(i) evidence of using scalar product on $\overrightarrow{\text{AB}}$ and $\overrightarrow{\text{AD}}$ (M1)

e.g. $\overrightarrow{\text{AB}} \cdot \overrightarrow{\text{AD}} = 5(1) + 2(3) + 1(2)$

$\overrightarrow{\text{AB}} \cdot \overrightarrow{\text{AD}} = 13 \quad \text{AI N2}$

(ii) $\left| \overrightarrow{\text{AB}} \right| = 5.477 \ldots$,

$\left| \overrightarrow{\text{AD}} \right| = 3.741 \ldots$ (A1)(A1)

evidence of using $\cos A = \frac{\overrightarrow{\text{AB}} \cdot \overrightarrow{\text{AD}}}{\left| \overrightarrow{\text{AB}} \right| \left| \overrightarrow{\text{AD}} \right|}$ (M1)

correct substitution $A1$

e.g. $\cos A = \frac{13}{20.493}$

$\widehat{A} = 0.884 \quad (50.6^\circ) \quad A1 \quad N3$

[7 marks]

Examiners report

Some candidates were unable to find the scalar product in part (c), yet still managed to find the correct angle, able to use the formula in the information booklet without knowing that the scalar product is a part of that formula.

126. Let $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. The vector $\mathbf{v} + p\mathbf{w}$ is perpendicular to $\mathbf{w}$. Find the value of $p$.

Markscheme

$p\mathbf{w} = p\mathbf{i} + 2p\mathbf{j} - 3p\mathbf{k}$ (seen anywhere) (A1)

try to find $\mathbf{v} + p\mathbf{w}$ (M1)

e.g. $3\mathbf{i} + 4\mathbf{j} + \mathbf{k} + p(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$

collecting terms $(3 + p)\mathbf{i} + (4 + 2p)\mathbf{j} + (1 - 3p)\mathbf{k} \quad A1$

attempt to find the dot product (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p)$

setting their dot product equal to 0 (M1)

e.g. $1(3 + p) + 2(4 + 2p) - 3(1 - 3p) = 0$

simplifying $A1$

e.g. $3 + p + 8 + 4p - 3 + 9p = 0$, $14p + 8 = 0$

$p = -0.571 \quad \text{AI N3}$

[7 marks]

Examiners report

This question was very poorly done with many leaving it blank. Of those that did attempt it, most were able to find $\mathbf{v} + p\mathbf{w}$ but really did not know how to proceed from there. They tried many approaches, such as, finding magnitudes, using negative reciprocals, or calculating the angle between two vectors. A few had the idea that the scalar product should equal zero but had trouble trying to set it up.
The point O has coordinates \((0, 0, 0)\), point A has coordinates \((1, -2, 3)\) and point B has coordinates \((-3, 4, 2)\).

127a. (i) Show that \(\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}\). [8 marks]

(ii) Find \(|\overrightarrow{BA}|\)\widehat{BA} and \(|\overrightarrow{OA}|\).

**Markscheme**

(i) evidence of approach \[ M1 \]

e.g. \(\overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{AB}\)

\(\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}\) \[ AG \ N0 \]

(ii) for choosing correct vectors, \((\overrightarrow{AO})\) with \((\overrightarrow{AB})\) or \((\overrightarrow{OA})\) with \((\overrightarrow{BA})\) \[ (A1)(A1) \]

Note: Using \((\overrightarrow{AO})\) with \((\overrightarrow{BA})\) will lead to \(\pi - 0.799\). If they then say \(|\overrightarrow{BA}|\widehat{BA} = 0.799\), this is a correct solution.

calculating \(\overrightarrow{AO} \bullet \overrightarrow{AB}\), \(|\overrightarrow{AO}|\) and \(|\overrightarrow{AB}|\) \[ (A1)(A1)(A1) \]

e.g. \(d_1 \bullet d_2 = (-1)(-4) + (2)(6) + (-3)(-1) = 19\)

\(|d_1| = \sqrt{(-1)^2 + 2^2 + (-3)^2} = \sqrt{14}\), \(|d_2| = \sqrt{(-4)^2 + 6^2 + (-1)^2} = \sqrt{53}\)

evidence of using the formula to find the angle \[ M1 \]

e.g. \(\cos \theta = \frac{-1 \cdot -4 + 2 \cdot 6 + -3 \cdot -1}{\sqrt{14} \sqrt{53}}\), \(0.69751\ldots\)

\(|\overrightarrow{BA}|\widehat{BA} = 0.799\) radians (accept \(45.8^\circ\)) \[ AI \ N3 \]

[8 marks]

**Examiners report**

Part (ai) was done well by most students. Most knew how to approach finding the angle in part (aii). The problems occurred when the incorrect vectors were chosen. If the vectors being used were stated, then follow through marks could be given.

127b. The line \(L_1\) has equation \(\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s\begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}\). Write down the coordinates of two points on \(L_1\). [2 marks]

**Markscheme**

two correct answers \[ A1A1 \]

e.g. \((1, -2, 3)\), \((-3, 4, 2)\), \((-7, 10, 1)\), \((-11, 16, 0)\) \[ N2 \]

[2 marks]

**Examiners report**

Part (b) was well done.

127c. The line \(L_2\) passes through \(A\) and is parallel to \(\overrightarrow{OB}\). [6 marks]

(i) Find a vector equation for \(L_2\), giving your answer in the form \(\mathbf{r} = \mathbf{a} + t\mathbf{b}\).

(ii) Point \(C(k, -k, 5)\) is on \(L_2\). Find the coordinates of \(C\).
Markscheme

(i) \( \mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} \)  \( A2 \) \( N2 \)

(ii) C on \( L_2 \), so \( \begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} \)  \( M1 \)

evidence of equating components  \( A1 \)

e.g. \( 1 - 3t = k \), \( -2 + 4t = -k \), \( 5 = 3 + 2t \)

one correct value \( t = 1 \), \( k = -2 \) (seen anywhere)  \( A1 \)

coordinates of \( C \) are \( (-2, 2, 5) \)  \( A1 \) \( N3 \)

[6 marks]

Examiners report
In part (ci), the error that occurred most often was the incorrect choice for the direction vector.

![Image](https://via.placeholder.com/150)

127d. The line \( L_3 \) has equation \( \mathbf{r} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \)  \( 2 \text{ marks} \)

Find the value of \( p \) at \( C \).

Markscheme

for setting up one (or more) correct equation using \( \mathbf{r} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \)  \( M1 \)

\( e.g. \) \( 3 + p = -2, -8 - 2p = 2, -p = 5 \)

\( p = -5 \)  \( A1 \) \( N2 \)

[2 marks]

Examiners report
Those that were able to find the coordinates in part (cii) were also able to be successful in part (d).

Consider the points \( A \ (1, 5, 4) \), \( B \ (3, 1, 2) \) and \( D \ (3, k, 2) \), with \( AD \perp AB \).

128a. Find

(i) \( \overrightarrow{AB} \) ;

(ii) \( \overrightarrow{AD} \) giving your answer in terms of \( k \).

[3 marks]
(i) evidence of combining vectors \( (M1) \)

\[
\overrightarrow{{\rm{AB}}} = \overrightarrow{{\rm{OB}}} - \overrightarrow{{\rm{OA}}}
\]

(or \( \overrightarrow{{\rm{AD}}} = \overrightarrow{{\rm{AO}}} + \overrightarrow{{\rm{OD}}} \) in part (ii))

\[
\overrightarrow{{\rm{AB}}} = \left( \begin{array}{c} 2 \\ -4 \\ -2 \end{array} \right)
\]

(A1) N2

(ii) \overrightarrow{{\rm{AD}}} = \left( \begin{array}{c} 2 \\ k - 5 \\ -2 \end{array} \right)

(A1) N1

[3 marks]

Examiners report

This question was well done by many candidates. Most found \( \overrightarrow{{\rm{AB}}} \) and \( \overrightarrow{{\rm{AD}}} \) correctly.

128b. Show that \( k = 7 \).

[3 marks]

Markscheme

evidence of using perpendicularity \( (M1) \)

\[
\left\langle \begin{array}{c} 2 \\ -4 \\ -2 \end{array} \right\rangle \bullet \left\langle \begin{array}{c} 2 \\ k - 5 \\ -2 \end{array} \right\rangle = 0
\]

\[
4 - 4(k - 5) + 4 = 0
\]

(A1)

\(-4k + 28 = 0\) (accept any correct equation clearly leading to \( k = 7 \)) (A1)

\( k = 7 \) AG N0

[3 marks]

Examiners report

The majority of candidates correctly used the scalar product to show \( k = 7 \).

128c. The point C is such that \( \overrightarrow{{\rm{BC}}} = \frac{1}{2}\overrightarrow{{\rm{AD}}} \).

Find the position vector of C.

[4 marks]

Markscheme

\[ \overrightarrow{{\rm{AD}}} = \left( \begin{array}{c} 2 \\ 2 \\ -2 \end{array} \right) \]

(A1)

\[ \overrightarrow{{\rm{BC}}} = \left( \begin{array}{c} 1 \\ 1 \\ -1 \end{array} \right) \]

(A1)

evidence of correct approach \( (M1) \)

\[
\left\langle \begin{array}{c} 3 \\ 1 \\ 2 \\ 2 \\ 2 \\ -2 \end{array} \right\rangle + \left\langle \begin{array}{c} 1 \\ 1 \\ -1 \end{array} \right\rangle = \left\langle \begin{array}{c} 4 \\ 2 \\ 1 \\ -1 \end{array} \right\rangle
\]

(A1) N3

[4 marks]

Examiners report

Some confusion arose in substituting \( k = 7 \) into \( \overrightarrow{{\rm{AD}}} \), but otherwise part (c) was well done, though finding the position vector of C presented greater difficulty.
128d. Find $\cos \widehat{\text{BAC}}$.

**Markscheme**

**METHOD 1**

choosing appropriate vectors, $\overrightarrow{\text{BA}}$, $\overrightarrow{\text{BC}}$ (A1)

finding the scalar product $\quad M1$

e.g. $-2(1) + 4(1) + 2(-1)$, $2(1) + (-4)(1) + (-2)(-1)$

$\cos \widehat{\text{BAC}} = 0$ (A1 N1)

**METHOD 2**

$\overrightarrow{\text{BC}}$ parallel to $\overrightarrow{\text{AD}}$ (may show this on a diagram with points labelled) $\quad R1$

$\overrightarrow{\text{BC}}$ \bot $\overrightarrow{\text{AB}}$ (may show this on a diagram with points labelled) $\quad R1$

$\cos \widehat{\text{BAC}} = 90^\circ$ (A1 N1)

[3 marks]

**Examiners report**

Owing to $\overrightarrow{\text{AB}}$ and $\overrightarrow{\text{BC}}$ being perpendicular, no problems were created by using these two vectors to find $\cos \widehat{\text{BAC}} = 0$, and the majority of candidates answering part (d) did exactly that.

129. The line $L_1$ is represented by $\begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line $L_2$ by $\begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$. The lines $L_1$ and $L_2$ intersect at point T. Find the coordinates of T.

**Markscheme**

evidence of equating vectors $\quad (M1)$

e.g. $[L_1] = [L_2]$

for any two correct equations $\quad A1A1$

e.g. $2 + s = 3 - t$, $5 + 2s = -3 + 3t$, $3 + 3s = 8 - 4t$

attempting to solve the equations $\quad (M1)$

finding one correct parameter $2 = -1$ $\quad A1$

the coordinates of T are $(1, 3, 0)$ $\quad A1 N3$

[6 marks]

**Examiners report**

Those candidates prepared in this topic area answered the question particularly well, often only making some calculation error when solving the resulting system of equations. Curiously, a few candidates found correct values for $s$ and $t$, but when substituting back into one of the vector equations, neglected to find the $z$-coordinate of T.

130. Two lines with equations $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

**Markscheme**

evidence of equating vectors $\quad (M1)$

e.g. $[L_1] = [L_2]$

for any two correct equations $\quad A1A1$

e.g. $2 + s = 3 - t$, $5 + 2s = -3 + 3t$, $3 + 3s = 8 - 4t$

attempting to solve the equations $\quad (M1)$

finding one correct parameter $2 = -1$ $\quad A1$

the coordinates of T are $(1, 3, 0)$ $\quad A1 N3$

[6 marks]

**Examiners report**

Those candidates prepared in this topic area answered the question particularly well, often only making some calculation error when solving the resulting system of equations. Curiously, a few candidates found correct values for $s$ and $t$, but when substituting back into one of the vector equations, neglected to find the $z$-coordinate of T.
Markscheme

evidence of appropriate approach \((M1)\)

\(\text{e.g. } (\begin{array}{c} 2 \\ 3 \\ -1 \end{array}) + s(\begin{array}{c} 5 \\ -3 \\ 2 \end{array}) = (\begin{array}{c} 9 \\ 2 \\ 2 \end{array}) + t(\begin{array}{c} -3 \\ 5 \\ -1 \end{array})\)

two correct equations \(\text{AI}(\text{AI})(\text{AI})\)

\(\text{e.g. } 2 + 5s = 9 - 3t, \ 3 - 3s = 2 + 5t, \ -1 + 2s = 2 - t\)

attempting to solve the equations \((M1)\)

one correct parameter \(s = 2, \ t = -1 \ \text{AI}\)

\(P\) is \((12, -3,3)\) (accept \((\begin{array}{c} 12 \\ -3 \\ 3 \end{array})\)) \(A1 \ N3\)

Examiners report

If this topic had been taught well then the candidates scored highly. The question was either well answered or not at all. Many candidates did not understand what was needed and tried to find the length of vectors or mid-points of lines. The other most common mistake was to use the values of the parameters to write the coordinates as \((\text{P}) = \left(8, -1\right)\).

131. Find the cosine of the angle between the two vectors \(3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}\) and \(4\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}\). \([6\ \text{marks}]\)

Markscheme

finding scalar product and magnitudes \((\text{AI})(\text{AI})(\text{AI})\)

\(\text{scalar product } = 12 - 20 - 15 = -23\)

\(\text{magnitudes } = \sqrt{3^2 + 4^2 + 5^2}, \ \sqrt{4^2 + (-5)^2 + (-3)^2}\)

\(\text{substitution into formula } \text{(M1)}\)

\(\text{e.g. } \cos \theta = \frac{(12 - 20 - 15)}{\sqrt{3^2 + 4^2 + 5^2} \times \sqrt{4^2 + (-5)^2 + (-3)^2}}\)

\(\cos \theta = -0.46 \ \text{AI} \ N4\)

Examiners report

Many candidates performed well in finding the magnitudes and scalar product to use the formula for angle between vectors. Some experienced trouble with the arithmetic to obtain the required result. A significant number of candidates isolated the \(\theta\) answering with \(\arccos\left(\frac{-23}{50}\right)\).

The line \(\{\mathbf{L}_1\}\) is parallel to the \(z\)-axis. The point \(P\) has position vector \((8, 1, 0)\) and lies on \(\{\mathbf{L}_1\}\).

132a. Write down the equation of \(\{\mathbf{L}_1\}\) in the form \(\mathbf{r} = \mathbf{a} + t\mathbf{b}\). \([2\ \text{marks}]\)

Markscheme

\(\{\mathbf{L}_1\} : \mathbf{r} = 8\mathbf{i} + 1\mathbf{j} + 0\mathbf{k} + t(0\mathbf{i} + 1\mathbf{j} + 0\mathbf{k})\) \(A2 \ N2\)

[2 marks]
Examiners report

Very few candidates gave a correct direction vector parallel to the $z$-axis. Provided they wrote down an equation here they were able to earn most of subsequent marks on follow through.

132b. The line $\mathcal{L}_2$ has equation $\mathbf{r} = \left( \begin{array}{c} 2 \\ 4 \\ -1 \end{array} \right) + s\left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right)$. The point A has position vector $\left( \begin{array}{c} 6 \\ 2 \\ 9 \end{array} \right)$. Show that A lies on $\mathcal{L}_2$.

Markscheme

evidence of equating $\mathbf{r}$ and $\overrightarrow{OA}$ (M1)
e.g. $\left( \begin{array}{c} 6 \\ 2 \\ 9 \end{array} \right) = \left( \begin{array}{c} 2 \\ 4 \\ -1 \end{array} \right) + s\left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right), A = r$
e.g. $6 = 2 + 2s, 2 = 4 - s, 9 = -1 + 5s$
s = 2 A1
evidence of confirming for other two equations A1
e.g. $6 = 2 + 4, 2 = 4 - 2, 9 = -1 + 10$
so A lies on $\mathcal{L}_2$ AG N0

Examiners report

For (b), many found the correct parameter but neglected to confirm it in the other two equations.

132c. Let B be the point of intersection of lines $\mathcal{L}_1$ and $\mathcal{L}_2$.

(i) Show that $\overrightarrow{OB} = \left( \begin{array}{c} 8 \\ 1 \\ 14 \end{array} \right)$.

(ii) Find $\overrightarrow{AB}$.
**Markscheme**

(i) evidence of approach  \( M1 \)

e.g. \(
\left( \begin{array}{c} 2 \\ 4 \\ -1 \end{array} \right) + s \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right)
\) = \( \left( \begin{array}{c} 8 \\ 1 \\ 0 \end{array} \right) + t \left( \begin{array}{c} 0 \\ 0 \\ 1 \end{array} \right) \)

one correct equation  \( A1 \)

e.g. \( 2 + 2s = 8 \), \( 4 - s = 1 \), \( -1 + 5s = t \)

attempt to solve  \( (M1) \)

finding \( s = 3 \)  \( A1 \)

substituting  \( M1 \)

e.g. \( \overrightarrow{OB} = \left( \begin{array}{c} 2 \\ 4 \\ -1 \end{array} \right) + 3 \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right) \)

\( \overrightarrow{OB} = \left( \begin{array}{c} 8 \\ 1 \\ 14 \end{array} \right) \)

(ii) evidence of appropriate approach  \( (M1) \)

e.g. \( \overrightarrow{AB} = \overrightarrow{DC} \)

correct values  \( A1 \)

e.g. \( \overrightarrow{OD} + \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \), \( \left( \begin{array}{c} x \\ y \\ z \end{array} \right) + \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \), \( \left( \begin{array}{c} 2 \\ -1 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 - x \\ 1 - y \\ -4 - z \end{array} \right) \)

\( \overrightarrow{OD} = \left( \begin{array}{c} 0 \\ 2 \\ -9 \end{array} \right) \)

[7 marks]

**Examiners report**

In (c) some performed a trial and error approach to obtaining an integer parameter and thus did not "show" the mathematical origin of the result. Finding vector \( \overrightarrow{AB} \) proved accessible.

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132d. The point C is at \((2, 1, -4)\). Let D be the point such that ABCD is a parallelogram.  \([3\; marks]\)

Find \( \overrightarrow{OD} \) .

**Markscheme**

evidence of appropriate approach  \( (M1) \)

e.g. \( \overrightarrow{AB} = \overrightarrow{DC} \)

correct values  \( A1 \)

e.g. \( \overrightarrow{OD} + \left( \begin{array}{c} 2 \\ -4 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \), \( \left( \begin{array}{c} x \\ y \\ z \end{array} \right) + \left( \begin{array}{c} 2 \\ -4 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 \\ 1 \\ -4 \end{array} \right) \), \( \left( \begin{array}{c} 2 \\ -4 \\ 5 \end{array} \right) = \left( \begin{array}{c} 2 - x \\ 1 - y \\ -4 - z \end{array} \right) \)

\( \overrightarrow{OD} = \left( \begin{array}{c} 0 \\ 2 \\ -9 \end{array} \right) \)

\([3\; marks]\)

**Examiners report**

A good number of candidates had an appropriate approach to (d), although surprisingly many subtracted \( \overrightarrow{OC} \) from \( \overrightarrow{AB} \) in finding \( \overrightarrow{OD} \) .
In the following diagram, $\overrightarrow{u} = \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{BD}$.

The midpoint of $\overrightarrow{AD}$ is $E$ and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of $\overrightarrow{u}$ and $\overrightarrow{v}$.

133a. $\overrightarrow{AE}$

Markscheme

$\overrightarrow{AE} = \frac{1}{2} \overrightarrow{AD}$ $\text{A1}$

Attempt to find $\overrightarrow{AD}$ $\text{M1}$

e.g. $\overrightarrow{AB} + \overrightarrow{BD}$

$\overrightarrow{AE} = \frac{1}{2}(u + v)$

$\left( = \frac{1}{2}u + \frac{1}{2}v \right)$ $\text{A1 N2}$

[3 marks]

Examiners report

[N/A]

133b. $\overrightarrow{EC}$

Markscheme

$\overrightarrow{EC} = \overrightarrow{AE} = \frac{1}{2}(u + v)$ $\text{A1}$

$\overrightarrow{DC} = 3v$ $\text{A1}$

Attempt to find $\overrightarrow{EC}$ $\text{M1}$

e.g. $\overrightarrow{ED} + \overrightarrow{DC}$

$\overrightarrow{EC} = \frac{1}{2}u + \frac{7}{2}v$

$\left( = \frac{1}{2}(u + 7v) \right)$ $\text{A1 N2}$

[4 marks]

Examiners report

[N/A]
Consider the lines \( L_1 \), \( L_2 \), \( L_3 \), and \( L_4 \), with respective equations.

\( L_1 \): \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
1 \\
2 \\
3
\end{pmatrix} + t\begin{pmatrix}
3 \\
-2 \\
1
\end{pmatrix}
\]

\( L_2 \): \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
1 \\
2 \\
3
\end{pmatrix} + p\begin{pmatrix}
3 \\
2 \\
1
\end{pmatrix}
\]

\( L_3 \): \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} + s\begin{pmatrix}
-1 \\
2 \\
-a
\end{pmatrix}
\]

\( L_4 \): \[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = q\begin{pmatrix}
-6 \\
4 \\
-2
\end{pmatrix}
\]

134a. Write down the line that is parallel to \( L_4 \). \( [1 \text{ mark}] \)

**Markscheme**

\( L_1 \) \( A1 \) \( N1 \)

\( [1 \text{ mark}] \)

**Examiners report**

[N/A]

134b. Write down the position vector of the point of intersection of \( L_1 \) and \( L_2 \). \( [1 \text{ mark}] \)

**Markscheme**

\( \begin{pmatrix}
1 \\
2 \\
3
\end{pmatrix} \) \( A1 \) \( N1 \)

\( [1 \text{ mark}] \)

**Examiners report**

[N/A]

134c. Given that \( L_1 \) is perpendicular to \( L_3 \), find the value of \( a \). \( [5 \text{ marks}] \)

**Markscheme**

choosing correct direction vectors \( A1A1 \)

e.g. \( \left( \begin{array}{c} 3 \\ -2 \\ 1 \end{array} \right), \left( \begin{array}{c} 1 \\ 0 \\ -a \end{array} \right) \) recognizing that \( \mathbf{a} \bullet \mathbf{b} = 0 \) \( M1 \)
correct substitution \( A1 \)
e.g. \(-3 - 4 - a = 0\)

\( a = -7 \) \( A1 \) \( N3 \)

\( [5 \text{ marks}] \)

**Examiners report**

[N/A]