SOLUCIONES MATEMATICAS II (PD) - 4ª EV (EXAMEN MOCK)- PRUEBA 2 (con CALCULADORA)

1) The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 . **[2+2+2]**

a) Find the common difference.

b) Find the 28^{th} term of the sequence.

c) Find the sum of the first 28 terms.

a)
e.g. subtracting terms, using sequence formula
$d = 1.7$ *A1* *N2*

b)
e.g. $5 + 27(1.7)$
28^{th} term is 50.9 (exact)

c)
e.g. $S_{28} = \frac{28}{2}(2(5) + 27(1.7))$, $\frac{28}{2}(5 + 50.9)$
$S_{28} = 782.6$ (exact) [782, 783]

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2) Consider the expansion of $\left(2x^3 + \frac{b}{x}\right)^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots$. **[3+3]**

a. Find b.

b. Find k.

a)
valid attempt to find term in x^{20}
e.g. $\binom{8}{1}(2^7)(b)$, $(2x^3)^7\left(\frac{b}{x}\right) = 3072$
correct equation
e.g. $\binom{8}{1}(2^7)(b) = 3072$
$b = 3$

b)
e.g. 7th term, $r = 6$
correct expression
e.g. $\binom{8}{6}(2x^3)^2\left(\frac{3}{x}\right)^6$
$k = 81648$ (accept 81600)

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3) Draw a draft of two functions that:

a)

$\lim\limits_{x \to -\infty} f(x) = -1$ $\quad \lim\limits_{x \to 3^-} f(x) = +\infty$

$\lim\limits_{x \to -2^-} f(x) = -\infty$ $\quad \lim\limits_{x \to 3^+} f(x) = -\infty$

$\lim\limits_{x \to -2^+} f(x) = +\infty$ $\quad \lim\limits_{x \to +\infty} f(x) = -1$

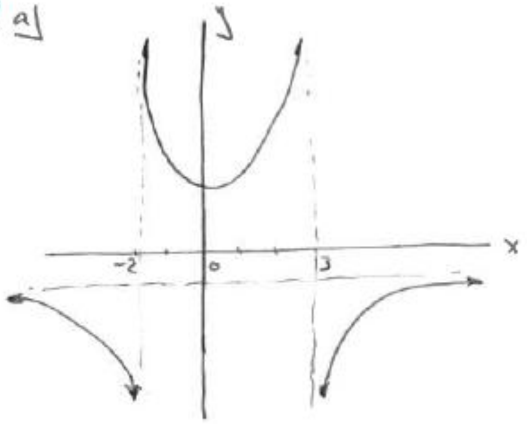
b)

$\lim\limits_{x \to 2^-} f(x) = -3$ $\quad \lim\limits_{x \to +\infty} f(x) = -\infty$

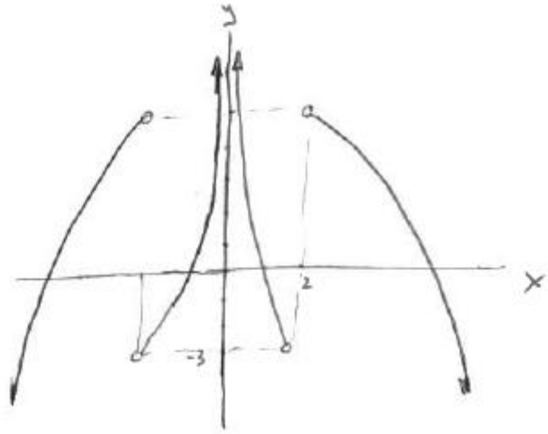
$\lim\limits_{x \to 2^+} f(x) = 5$ $\quad f(x)$ OY-symmetric

$\lim\limits_{x \to 0^+} f(x) = +\infty$

3) a)



b)



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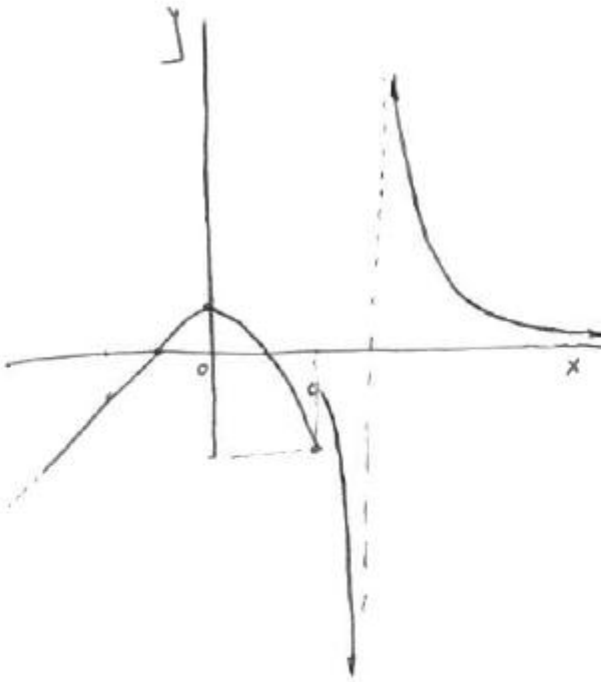
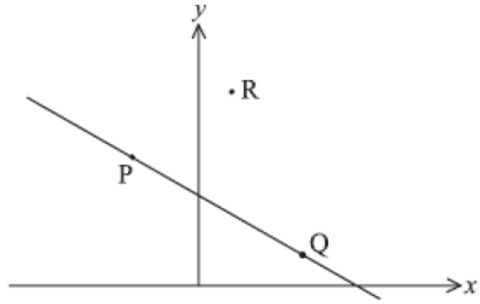
4. Obtain the Domain and draft the grapfic: $f(x) = \begin{cases} x+1 & \text{Si } x \le -1 \\ 1-x^2 & \text{Si } -1 < x \le 2 \\ \dfrac{1}{x-3} & \text{Si } x > 2 \end{cases}$

4. $x-3=0 \rightarrow x=3$ $\quad dom\, f = (-\infty,-1] \cup (-1,2] \cup (2,+\infty) - \{3\} = \boxed{\mathbb{R} - \{3\}}$



5) En el diagrama siguiente se muestran los puntos P(−2, 4), Q(3, 1) y R(1, 6).



(a) Halle el vector $\vec{PQ}$.

(b) Halle una ecuación vectorial de la recta que pasa por R y es paralela a la recta (PQ).

5) a) $\vec{PQ} = \vec{OQ} - \vec{OP} = \begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}5\\-3\end{pmatrix}$

b) $\boxed{\vec{r} = \begin{pmatrix}1\\6\end{pmatrix} + t\cdot\begin{pmatrix}5\\-3\end{pmatrix}}$

6) La siguiente figura muestra un círculo de centro O y radio 12 cm. La cuerda AB determina un ángulo central de 75º. Las tangentes a la circunferencia en A y en B se encuentran en P.

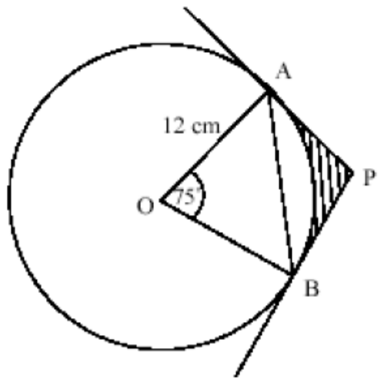
a) Halla el área del sector OAB

b) Halla el área del triángulo OAB

c) Demuestra que $AB = 12\sqrt{2(1-\cos 75^\circ)}$

d) Halla el área del triángulo ABP

e) Halla el área de la región sombreada



a) $75^\circ = \dfrac{75\cdot\pi}{180} = \dfrac{5\pi}{12}\,rad \rightarrow Area\ Sector = \dfrac{12^2\cdot 5\pi/12}{2} = \boxed{30\pi\ cm^2}$

b) $Area\ Triángulo\ OAB = \dfrac{12\cdot 12\cdot sen75^\circ}{2} \approx \boxed{69{,}5\ cm^2}$

c) $AB = \sqrt{12^2+12^2-2\cdot 12\cdot 12\cdot\cos 75^\circ} = \sqrt{12^2(1+1-2\cdot\cos 75^\circ)} = 12\sqrt{2(1-\cos 75^\circ)} \approx \boxed{14{,}6cm}$

d) $180^\circ - 75^\circ = 105^\circ \rightarrow O\hat{A}B = O\hat{B}A = \dfrac{105}{2} = 52{,}5^\circ \rightarrow B\hat{A}P = A\hat{B}P = 90^\circ - 52{,}5^\circ = 37{,}5^\circ \rightarrow A\hat{P}B = 180^\circ - 2\cdot 37{,}5^\circ = 105^\circ$

$AB^2 = AP^2 + PB^2 - 2\cdot AP\cdot PB\cdot\cos 105^\circ$

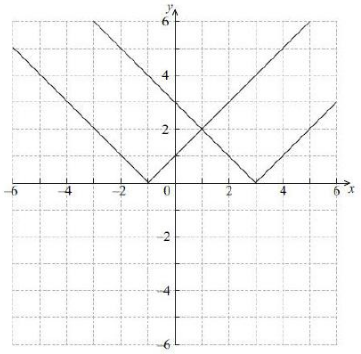
Como $AP = PB \rightarrow AB^2 = AP^2 + AP^2 - 2\cdot AP\cdot AP\cdot\cos 105^\circ \rightarrow AB^2 = AP^2(2-2\cdot\cos 105^\circ) \rightarrow$

$\rightarrow AP^2 = \dfrac{AB^2}{2-2\cdot\cos 105^\circ} \Rightarrow AP = \dfrac{AB}{\sqrt{2-2\cdot\cos 105^\circ}} = \dfrac{14{,}6}{\sqrt{2-2\cdot\cos 105^\circ}} \approx 9{,}20\,cm$

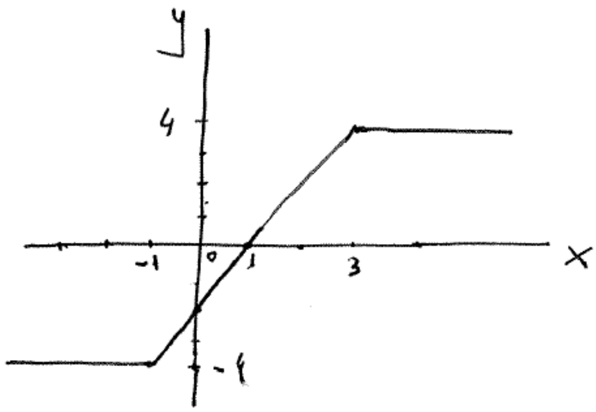
$Area\ Triángulo\ ABP = \dfrac{AB\cdot AP\cdot senB\hat{A}P}{2} = \dfrac{14{,}6\cdot 9{,}20\cdot sen37{,}5^\circ}{2} \approx \boxed{40{,}9\ cm^2}$

e) $Area\ Sombreada = Area\ Triángulo\ OAB + Area\ Triángulo\ ABP - Area\ Sector \approx \boxed{16{,}2\ cm^2}$

7) The graphs of $y=|x+1|$ and $y=|x-3|$ are shown below.



Let $f(x)=|x+1|-|x-3|$. Draw the graph of $y=f(x)$ on the same grid



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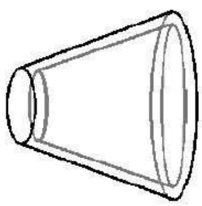
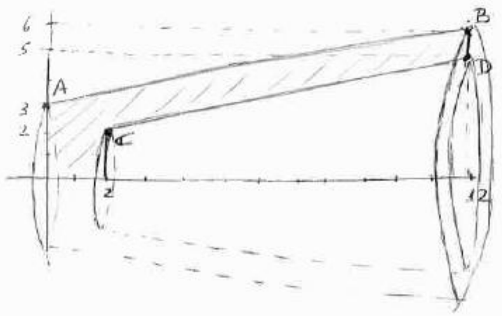
8. Calcula los cm^3 necesarios para construir el vaso de la figura.
Radios Interiores: 2 cm. y 5 cm.
Radios Exteriores: 3 cm. y 6 cm.
Altura Interior: 10 cm
Altura Exterior: 12 cm



8)



$A(0,3)$, $B(12,6)$ $\rightarrow$ $\vec{AB}=(12,3)$ $\rightarrow$ pend $=\frac{3}{12}=\frac{1}{4}$

$y-3=\frac{1}{4}(x-0) \rightarrow \boxed{y=\frac{x}{4}+3}$

$C(2,2)$, $D(12,5)$ $\rightarrow$ $\vec{CD}=(10,3)$ $\rightarrow$ pend $=\frac{3}{10}$

$y-2=\frac{3}{10}(x-2) \rightarrow y=\frac{3x}{10}-\frac{6}{10}+2$

$\boxed{y=\frac{3x+14}{10}}$

$$V=\pi\int_0^2\left(\frac{x}{4}+3\right)^2dx+\pi\int_2^{12}\left[\left(\frac{x}{4}+3\right)^2-\left(\frac{3x+14}{10}\right)^2\right]dx=$$

$$=\pi\int_0^{12}\left(\frac{x}{4}+3\right)^2dx-\pi\int_2^{12}\left(\frac{3x+14}{10}\right)^2dx=\pi\,4\,\frac{\left(\frac{x}{4}+3\right)^3}{3}\Bigg|_0^{12}-\pi\frac{10}{3}\,\frac{\left(\frac{3x+14}{10}\right)^3}{3}\Bigg|_2^{12}=$$

$$=288\pi-\frac{1250\pi}{9}+\frac{80\pi}{9}=\frac{1422\pi}{9}=\boxed{158\pi \ \text{u.v}}$$

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9) A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t}, \; t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

$v = 50 - 50\, e^{-0'2t}$

a) $t=0 \Rightarrow v = 50 - 50 \cdot e^{0} = \boxed{0 \text{ m/sg}}$

$t=10 \Rightarrow v = 50 - 50\, e^{-2} = \boxed{43,2 \text{ m/sg}}$

b) $a = \dfrac{dv}{dt} = -50 \cdot (-0'2)\, e^{-0'2t} = \boxed{10\, e^{-0'2t}}$

$t=0 \Rightarrow a = 10 \cdot e^{0} = \boxed{10 \text{ m/sg}^2}$

c) $\lim\limits_{t \to +\infty} v = \lim\limits_{t \to +\infty} (50 - 50e^{-0'2t}) = 50 - 50 \cdot e^{-\infty} = 50 - 50 \cdot 0 = \boxed{50 \text{ m/sg}}$

$\lim\limits_{t \to +\infty} a = \lim\limits_{t \to +\infty} 10\, e^{-0'2t} = 10 \cdot e^{-\infty} = 10 \cdot 0 = \boxed{0 \text{ m/sg}^2}$

Con el paso del tiempo, la velocidad aumenta debido a que dispone de aceleración, pero como ésta se hace cada vez menor, el crecimiento de la velocidad es cada vez menor estabilizándose en $t \to +\infty$ en 50 m/sg.

d) $y = \int v\,dt = \int (50 - 50\, e^{-0'2t})\, dt = 50t - 50 \frac{1}{-0'2} e^{-0'2t} + K = \boxed{50t + 250e^{-0'2t} + K}$ ✓

$\left.\begin{matrix} t=0 \\ y=0 \end{matrix}\right| \Rightarrow 0 = 50 \cdot 0 + 250 \cdot e^{0} + K \;;\; 0 = 250 + K \Rightarrow \boxed{K = -250}$

$y = 250 \text{ m} \rightarrow \quad 250 = 50t + 250\, e^{-0'2t} - 250 \;;\; 500 = 50t + 250\, e^{-0'2t}$;

$10 = t + 5e^{-0'2t}$

$\boxed{t = 9'2070 \text{ sg}}$

10) Bag A contains 2 red and 3 green balls.

(a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

Bag B contains 4 red and n green balls.

(b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\frac{2}{15}$, show that $n = 6$.

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

a) $\boxed{\begin{matrix}2R\\3V\end{matrix}}$ $\quad P(R_1 \cap R_2) = \frac{2}{5}\cdot\frac{1}{4} = \frac{2}{20} = \boxed{0'1}$

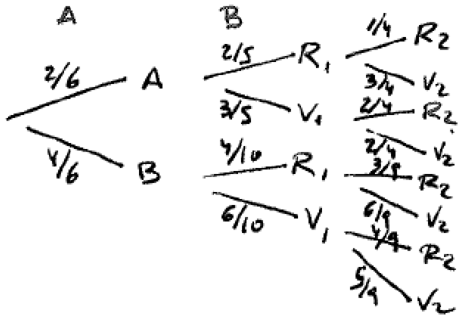
b) $\boxed{\begin{matrix}4R\\ 'n'\ V\end{matrix}}$ $\quad P(R_1 \cap R_2) = \frac{4}{n+4}\cdot\frac{3}{n+3} = \frac{12}{n^2+7n+12}$

$P(R_1 \cap R_2) = \frac{2}{15} \Rightarrow \frac{2}{15} = \frac{12}{n^2+7n+12}$; $2n^2+14n+24 = 180$;

$n^2+7n-78=0$; $n = \frac{-7\pm\sqrt{49+312}}{2} = \frac{-7\pm 19}{2} \rightarrow 6$ ✓ $\rightarrow \cancel{-13}$

c) $\boxed{\begin{matrix}2R\\3V\end{matrix}}$ $\boxed{\begin{matrix}4R\\6V\end{matrix}}$

A B



$P(R_1 \cap R_2) = \frac{2}{6}\cdot\frac{2}{5}\cdot\frac{1}{4} + \frac{4}{6}\cdot\frac{4}{10}\cdot\frac{3}{9} = \boxed{\frac{11}{90}}$

d) $P(A / R_1 \cap R_2) = \frac{P(A \cap R_1 \cap R_2)}{P(R_1 \cap R_2)} = \frac{\frac{2}{6}\cdot\frac{2}{5}\cdot\frac{1}{4}}{\frac{11}{90}} = \boxed{\frac{3}{11}}$

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11) The weights of adult males of a type of dog may be assumed to be normally distributed with mean 25 kg and standard deviation 3 kg. Given that 30 % of the weights lie between 25 kg and x kg, where $x > 25$, find the value of x.

$N(25; 3)$

$P[25 < X < x] = 30\%$

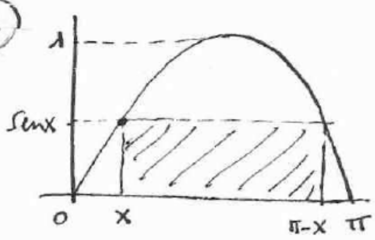
$P[X < x] = 80\% \Rightarrow x = \boxed{27'52}$ Kg. Hecho con calculadora gráfica en $N(25,3)$

12 Se dibuja un rectángulo cuyos vértices inferiores se encuentran en el eje OX y cuyos vértices superiores se encuentran en la curva $y = senx$, siendo $0 \leq x \leq \pi$

a) Escriba una expresión para el área del rectángulo

b) Halle el área máxima del rectángulo

12



$$\text{Area} = (\pi - 2x)\,\text{sen}x \qquad x \in (0, \pi/2)$$

$$\frac{dA}{dx} = -2\,\text{sen}x + (\pi - 2x)\cos x$$

$$\frac{dA}{dx} = 0 \rightarrow -2\,\text{sen}x + (\pi - 2x)\cos x = 0 \rightarrow$$

$$\rightarrow 2\,\text{sen}x = (\pi - 2x)\cos x \rightarrow 2\,\text{tg}x = \pi - 2x \Rightarrow$$

Resuelta con Calculadora gráfica $\boxed{x = 0'710246}$

$$\text{Area Máxima} = (\pi - 2 \cdot 0'71)\,\text{sen}\,0'71 = \boxed{0'021}$$