SOLUCIONES MODELO MATEMATICAS II (PD) - 4ª EV - PRUEBA 1 (SIN CALCULADORA)

1. **Find the "General term":** **[6 Marks]**

a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ b) $\frac{2}{1}, \frac{4}{3}, \frac{6}{5}, \frac{8}{7}, \frac{10}{9}, \ldots$ c) $1, 2, 6, 24, 120, \ldots$

d) $\frac{3}{3}, \frac{9}{6}, \frac{27}{9}, \frac{81}{12}, \ldots$ e) $\frac{3}{1}, -\frac{5}{4}, \frac{7}{9}, -\frac{9}{16}, \frac{11}{25}, \ldots$ e) $1, 4, 16, 64, \ldots$

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a) $a_n = \frac{n}{n+1}$ b) $a_n = \frac{2n}{2n-1}$ c) $a_n = n!$

d) $a_n = \frac{3^n}{3n}$ e) $a_n = (-1)^{n+1} \cdot \frac{2n+1}{n^2}$ f) $a_n = 4^{n-1}$

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2) Find an expression for the sum of the first 35 terms of the series

$$\ln x^2 + \ln\frac{x^2}{y} + \ln\frac{x^2}{y^2} + \ln\frac{x^2}{y^3} + \ldots$$

giving your answer in the form $\ln\frac{x^m}{y^n}$, where $m, n \in \mathbb{N}$.

2) $$\ln x^2 + \ln\frac{x^2}{y} + \ln\frac{x^2}{y^2} + \cdots + \ln\frac{x^2}{y^{34}} = \ln\left[x^2 \cdot \frac{x^2}{y} \cdot \frac{x^2}{y^2} \cdots \frac{x^2}{y^{34}}\right] =$$

$$= \ln\left(\frac{(x^2)^{35}}{y^{1+2+\cdots+34}}\right) = \ln\left(\frac{x^{70}}{y^{\frac{1+34}{2}\cdot 34}}\right) = \boxed{\ln\frac{x^{70}}{y^{595}}}$$

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3) Halla la primera y la segunda derivadas de: $y = \frac{x^2 - x}{(2x-1)^2}$

3) $$y' = \frac{\cancel{(2x-1)}(2x-1)^2 - (x^2-x)\cdot 2\cancel{(2x-1)}\cdot 2}{(2x-1)^{\cancel{4}\,3}} = \frac{(2x-1)^2 - 4(x^2-x)}{(2x-1)^3} = \frac{4x^2-4x+1-4x^2+4x}{(2x-1)^3} = \boxed{\frac{1}{(2x-1)^3}}$$

$$y'' = \frac{-1\cdot 3(2x-1)^{\cancel{2}}\cdot 2}{(2x-1)^{\cancel{6}\,4}} = \boxed{\frac{-6}{(2x-1)^4}}$$

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4) Resuelve: a) $5^{2x}-30\cdot5^{x}+125=0$ b) $9^{x}-2\cdot3^{x+2}+81=0$ c) $2^{x-1}+2^{x}+2^{x+1}=7$ **[3+3+3]**

4) a) $5^{2x}-30\cdot5^{x}+125=0$

$t=5^x$ $\quad t^2-30t+125=0\ ;\ t=\begin{cases}25\to 5^x=25 & \boxed{x=2}\\ 5\to 5^x=5 & \boxed{x=1}\end{cases}$

b) $9^x-2\cdot3^{x+2}+81=0$

$3^{2x}-2\cdot3^x\cdot3^2+81=0$

$t=3^x$ $\quad t^2-18t+81=0\ ;\ t=\begin{cases}9\\9\end{cases}\to 3^x=9 \quad \boxed{x=2}$

c) $2^{x-1}+2^x+2^{x+1}=7$

$\frac{2^x}{2}+2^x+2\cdot2^x=7$

$t=2^x$ $\quad \frac{t}{2}+t+2t=7\ ;\ t+2t+4t=14\ ;\ 7t=14\ ;\ t=2\to2^x=2 \quad \boxed{x=1}$

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5. Solve the following equations a) $\dfrac{x+1}{x+2}+\dfrac{x-1}{x-2}=\dfrac{2x+1}{x+1}$ b) $\dfrac{x^2+2}{x+1}-\dfrac{3-3x}{x-1}=\dfrac{7x+1}{x^2-1}$

5) a) $\dfrac{x+1}{x+2}+\dfrac{x-1}{x-2}=\dfrac{2x+1}{x+1}$ $\quad mcm=(x+2)(x-2)(x+1)$

$(x+1)(x-2)(x+1)+(x-1)(x+2)(x+1)=(2x+1)(x+2)(x-2)$

$(x^2-x-2)(x+1)+(x^2-1)(x+2)=(2x+1)(x^2-4)$

$x^3+\cancel{x^2}-\cancel{x^2}-x-2x-2+x^3+2x^2-x-2=2x^3-8x+x^2-4$

$x^2+4x=0$

$\boxed{x=\begin{cases}0\\-4\end{cases}}$

b) $\dfrac{x^2+2}{x+1}-\dfrac{3-3x}{x-1}=\dfrac{7x+1}{x^2-1}$ $\quad mcm=x^2-1$

$(x^2+2)(x-1)-(3-3x)(x+1)=7x+1$

$x^3-x^2+2x-2-\cancel{3x}-3+3x^2+\cancel{3x}=7x+1$

$x^3+2x^2-5x-6=0$

	1	2	-5	-6
-1		-1	-1	6
	1	1	-6	0
2		2	6	
	1	3	0	
-3		-3		
	1	0		

Soluciones: $\cancel{x=-1}$ Se anulan dos denominadores
$x=2$
$x=-3$

6. Siendo $\sqrt[n]{2^m \cdot 3^p} = \sqrt[3]{36} \cdot \sqrt[4]{24}$, halla los números enteros ***m***, ***n*** y ***p*** **[5]**

(6) $\sqrt[3]{36} \cdot \sqrt[4]{24} = \sqrt[3]{3^2 \cdot 2^2} \cdot \sqrt[4]{2^3 \cdot 3} = (3^2 \cdot 2^2)^{1/3} \cdot (2^3 \cdot 3)^{1/4} = (3^2 \cdot 2^2)^{4/12} \cdot (2^3 \cdot 3)^{3/12} =$

$= \sqrt[12]{(3^2 \cdot 2^2)^4 \cdot (2^3 \cdot 3)^3} = \sqrt[12]{3^8 \cdot 2^8 \cdot 2^9 \cdot 3^3} = \sqrt[12]{2^{17} \cdot 3^{11}}$

$\boxed{m = 17 \quad n = 12 \quad p = 11}$

7) The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$. Find the possible values of a

7) $\left(x + \frac{1}{ax^2}\right)^7 = \cdots\cdots + \binom{7}{2} x^5 \cdot \left(\frac{1}{ax^2}\right)^2 + \cdots\cdots = \cdots\cdots + \frac{21}{a^2} x + \cdots\cdots$

$\frac{21}{a^2} = \frac{7}{3} \Rightarrow \boxed{a = \pm 3}$

8) Halla las ecuaciones de las dos rectas tangentes a la curva: $y = \frac{2x+5}{x+3}$ que tienen de pendiente 1/9.

8) $y' = \frac{2(x+3) - (2x+5)}{(x+3)^2} = \frac{1}{(x+3)^2}$

$y' = \frac{1}{9} \Rightarrow \frac{1}{9} = \frac{1}{(x+3)^2} \Rightarrow (x+3)^2 = 9 \Rightarrow x+3 = \pm 3 \begin{cases} x = 0 \\ x = -6 \end{cases}$

$x = 0 \begin{cases} y = \frac{5}{3} \\ y' = \frac{1}{9} \end{cases}$ $\boxed{y - \frac{5}{3} = \frac{1}{9} x}$

$x = -6 \begin{cases} y = -\frac{7}{3} \\ y' = \frac{1}{9} \end{cases}$ $\boxed{y + \frac{7}{3} = \frac{1}{9}(x+6)}$

9. a) Demuestre que $\frac{sen(2\alpha)}{1+\cos(2\alpha)} = tg\alpha$

b) Partiendo de aquí, halle el valor de $ctg(\pi/8)$ en la forma $a + b\sqrt{2}$ con $a, b \in Z$.

(9)

a) $\frac{sen(2\alpha)}{1+\cos(2\alpha)} = \frac{2 sen\alpha \cos\alpha}{1+\cos^2\alpha - sen^2\alpha} = \frac{2 sen\alpha \cos\alpha}{\cancel{1}+\cos^2\alpha - \cancel{1} + \cos^2\alpha} = \frac{2 sen\alpha \cos\alpha}{2\cos^2\alpha} = tg\alpha$

b) $ctg(\pi/8) = \frac{1}{tg(\pi/8)} = \frac{1}{\frac{sen(2\pi/8)}{1+\cos(2\pi/8)}} = \frac{1+\cos(\pi/4)}{sen(\pi/4)} = \frac{1+\sqrt{2}/2}{\sqrt{2}/2} = \frac{2+\sqrt{2}}{\sqrt{2}} =$

$= \frac{(2+\sqrt{2})\sqrt{2}}{(\sqrt{2})^2} = \frac{2\sqrt{2}+2}{2} = \boxed{1+\sqrt{2}}$ $a = 1$, $b = 1$

10) Let $f : x \mapsto \sqrt{\dfrac{1}{x^2} - 2}$. Find

(a) the set of real values of x for which f is real and finite ;

(b) the range of f.

10) $f(x) = \sqrt{\frac{1}{x^2} - 2}$

$\frac{1}{x^2} - 2 \geq 0$; $x \neq 0$

$\frac{1}{x^2} - 2 = 0$; $\frac{1}{x^2} = 2$; $\frac{1}{2} = x^2$; $x = \pm\sqrt{\frac{1}{2}}$

	$-\infty$ $-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$ 0	0 $+\sqrt{\frac{1}{2}}$	$+\sqrt{\frac{1}{2}}$ $+\infty$
Signo de $\frac{1}{x^2} - 2$	$-$	$+$	$+$	$-$

a) $\boxed{\operatorname{dom} f = \left[-\sqrt{\frac{1}{2}}, +\sqrt{\frac{1}{2}}\right] - \{0\}}$

b)
- $f(x)$ es siempre ≥ 0
- $f(x) = 0$ para $x = \pm\sqrt{\frac{1}{2}}$
- $\lim_{x \to 0} f(x) = \sqrt{+\infty - 2} = +\infty$

$\Rightarrow \boxed{\operatorname{im} f = [0, +\infty)}$

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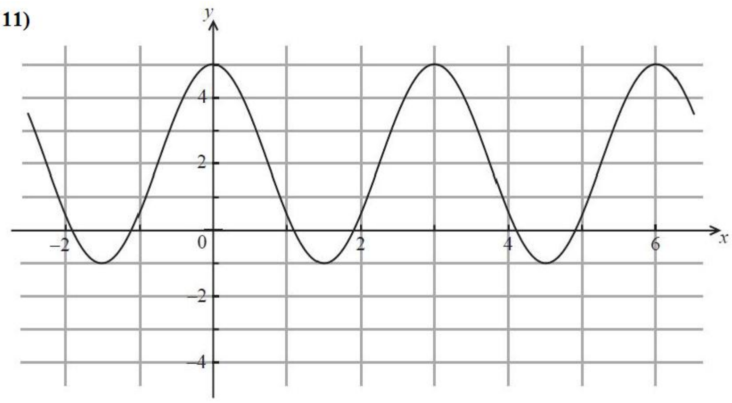
11)

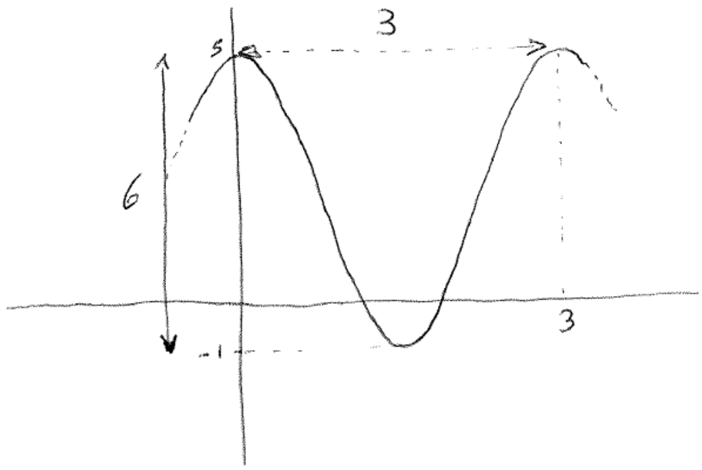


Find the value of a, the value of b and the value of c. **[6]**

11)

$y = a\cos(bx) + c$

- La amplitud es: $5 - (-1) = 6$
  Por lo tanto $\boxed{a = 3}$
- Los máximos están a altura 5.
  Por lo tanto $3 + c = 5 \Rightarrow \boxed{c = 2}$
- La diferencia de abscisas entre máximos es $3 \Rightarrow$ Periodo $= 3$
  Por lo tanto: $b \cdot 3 = 2\pi \Rightarrow \boxed{b = \frac{2\pi}{3}}$



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12) Obtain the value of the area enclosed by the following functions:

$f(x) = x^4 - 5x^2 + 4; \quad x = -3; \quad x = 3$ and the X-axis

12) $y = x^4 - 5x^2 + 4$

$y = 0 \Rightarrow x^4 - 5x^2 + 4 = 0 \Rightarrow x^2 = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \begin{cases} 4 \Rightarrow x = \pm 2 \\ 1 \Rightarrow x = \pm 1 \end{cases}$

$\begin{array}{ccccccccc} + & & - & & + & & - & & + \\ & -2 & & -1 & & 1 & & 2 & \end{array}$

signo $x^4 - 5x^2 + 4$

$$\text{Area} = \int_{-3}^{3} |x^4 - 5x^2 + 4|\, dx = \int_{-3}^{-2} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{1} f(x)\,dx - \int_{1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx =$$

$$= 2\int_{0}^{1} f(x)\,dx - 2\int_{1}^{2} f(x)\,dx + 2\int_{2}^{3} f(x)\,dx =$$

$$= 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_0^1 - 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_1^2 + 2\left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right)\Big|_2^3 =$$

$$= 2\left(\frac{1}{5} - \frac{5}{3} + 4\right) - 2\left(\frac{32}{5} - \frac{40}{3} + 8\right) + 2\left(\frac{1}{5} - \frac{5}{3} + 4\right) + 2\left(\frac{243}{5} - \frac{135}{3} + 12\right) - 2\left(\frac{32}{5} - \frac{40}{3} + \right.$$

$$= 4\left(\frac{1}{5} - \frac{5}{3} + 4\right) - 4\left(\frac{32}{5} - \frac{40}{3} + 8\right) + 2\left(\frac{243}{5} - \frac{135}{3} + 12\right) =$$

$$= \frac{4 - 128 + 486}{5} + \frac{160 - 20 - 270}{3} + 16 - 32 + 24 = \frac{362}{5} - \frac{130}{3} + 8 =$$

$$= \frac{1086 - 650 + 120}{15} = \boxed{\frac{556}{15}\ \text{us}}$$

13) La distribución de probabilidad de una variable aleatoria discreta X viene dada por

$$\mathrm{P}(X = x) = cx(5 - x),\ x = 1;\ 2;\ 3;\ 4.$$

(a) Halle el valor de c.

(b) Halle $\mathrm{E}(X)$.

$P[X = x] = cx(5-x) \qquad x = 1, 2, 3, 4.$

a) $\begin{array}{c|cccc} x & 1 & 2 & 3 & 4 \\ \hline P & 4c & 6c & 6c & 4c \end{array} \Rightarrow 4c + 6c + 6c + 4c = 1 \ ; \ 20c = 1 \Rightarrow \boxed{c = \frac{1}{20}}$

b) $E[x] = \sum_{x=1}^{4} x \cdot P[x = x] = 1 \cdot \frac{4}{20} + 2 \cdot \frac{6}{20} + 3 \cdot \frac{6}{20} + 4 \cdot \frac{4}{20} = \frac{50}{20} = \boxed{2'5}$

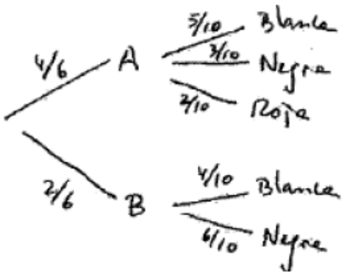
14) Dos urnas A y B, que contienen bolas de colores, tiene la siguiente composición:

Urna A: 5 blancas, 3 negras y 2 rojas. Urna B: 4 blancas y 6 negras.

También tenemos un dado que tiene 4 caras marcadas con la letra A y las otras dos con la letra B. Tiramos el dado y sacamos una bola al azar de la urna que indica el dado

a) ¿Cuál es la probabilidad de que esa bola sea blanca?

b) ¿Cuál es la probabilidad de que esa bola no sea roja?



$$P(\text{Blanca}) = \frac{4}{6}\cdot\frac{5}{10} + \frac{2}{6}\cdot\frac{4}{10} = \frac{28}{60} = \boxed{\frac{7}{15}}$$

$$P(\overline{\text{Roja}}) = 1 - P(\text{Roja}) = 1 - \frac{4}{6}\cdot\frac{2}{10} = 1 - \frac{8}{60} = \frac{52}{60} = \boxed{\frac{13}{15}}$$

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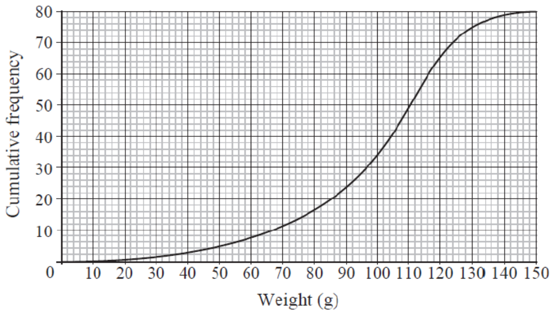
15) The cumulative frequency graph below represents the weight in grams of 80 apples picked from a particular tree.



(a) Estimate the

(i) median weight of the apples;

(ii) 30th percentile of the weight of the apples.

(b) Estimate the number of apples which weigh more than 110 grams.

a) $\frac{80}{2} = 40 \Rightarrow \boxed{\text{mediana} = 104\text{ g}}$

$30\% \cdot 80 = 24 \Rightarrow \boxed{P_{30} = 90\text{ g}}$

b) $x = 110 \rightarrow F = 48$

$80 - 48 = \boxed{32 \text{ manzanas}}$