JUN 94

i) rango = nº de filas (o columnas) de una matriz que resulten ser linealmente independientes

ii) rango $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3 \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \neq 0$

Si eliminamos por ejemplo la 1ª columna, quedaría $\begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix}$.

Si rango $\begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix} \leq 1 \Rightarrow \begin{matrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} = 0 \\ \begin{vmatrix} b & c \\ h & i \end{vmatrix} = 0 \\ \begin{vmatrix} e & f \\ h & i \end{vmatrix} = 0 \end{matrix} \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - d \begin{vmatrix} b & c \\ h & i \end{vmatrix} + g \cdot \begin{vmatrix} b & c \\ e & f \end{vmatrix} = 0$ Absurdo.

Luego: $\boxed{\text{rango} \begin{pmatrix} b & c \\ e & f \\ h & i \end{pmatrix} = 2}$

Si eliminamos por ejemplo la 1ª fila y la 1ª columna: $\begin{pmatrix} e & f \\ h & i \end{pmatrix}$

Podría ser de rango 1: Por ejemplo, $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 1 \\ 5 & 1 & 1 \end{pmatrix}$ tiene rango 3

mientras que $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ tiene rango 1.

SEPT 94

$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 2-x & 1 \\ 1 & 1 & 1 & 3-x \end{vmatrix}$ contiene sólo tres veces la variable $x$ y se multiplican entre sí solo una vez al no repetir ni fila ni columna, por lo que $1 \cdot (1-x)(2-x)(3-x)$ daría un término en $x^3$ que no podría simplificarse con ningún otro, luego su grado es **3**. La ecuación $f(x)=0$ tendría entonces 3 soluciones.

Vemos que para $x=0$ se igualan las dos primeras filas, dando el determinante nulo, por lo que cumpliría $f(x)=0$

Lo mismo sucede con $x=1$ (1ª fila = 3ª fila)

y con $x=2$ (1ª fila = 4ª fila)

Por lo tanto las 3 soluciones son $\boxed{\begin{matrix} x=0 \\ x=1 \\ x=2 \end{matrix}}$

JUN 95

a) $A \cdot C \cdot B$

$(m \times n) \cdot (q \times r) \cdot (n \times p) = (m \times p)$ es el orden del resultado (de poder hacerse)

Debe cumplirse $\left. \begin{matrix} n = q \\ r = n \end{matrix} \right\} \rightarrow \boxed{n = q = r}$

b) $A \cdot (B + C)$

$(m \times n) \cdot [(n \times p) + (q \times r)] = \begin{matrix} (m \times p) \\ (m \times r) \end{matrix}$ es el orden del resultado (de poder hacerse)

Debe cumplirse: $\boxed{\begin{matrix} n = q \\ p = r \end{matrix}}$

c)
$$\begin{pmatrix}3&4\\1&2\end{pmatrix}A=\begin{pmatrix}3&4\\1&2\end{pmatrix}B \Rightarrow A=\begin{pmatrix}3&4\\1&2\end{pmatrix}^{-1}\cdot\begin{pmatrix}3&4\\1&2\end{pmatrix}B = IB = B \Rightarrow \boxed{A=B}$$

$\begin{vmatrix}3&4\\1&2\end{vmatrix}=6-4=2\neq 0$, por lo que existe su inversa.

JUN 95

i) rango = nº filas (o columnas) que forman un conjunto linealmente independ.

conjunto linealmente independiente (de filas) = ninguna fila se puede generar con una combinación lineal de las restantes filas del conjunto.

Combinación lineal (de filas) = suma de filas multiplicadas por constantes.

ii)
$\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}=0$ y existe algún menor de orden 2 que es $\neq 0$.
es decir que las 3 columnas no son proporcionales.

$$\begin{vmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{vmatrix}=\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}+\begin{vmatrix}a&b&a\\d&e&d\\g&h&g\end{vmatrix}=0+0=0$$

columnas iguales

Luego el rango de $\begin{pmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{pmatrix}$ no será 3

Veamos que no puede ser menor de 2.

Si no alcanza el rango es que sus menores de orden 2 son todos nulos:
$\begin{vmatrix}a&b\\d&e\end{vmatrix}=0$, $\begin{vmatrix}a&b\\g&h\end{vmatrix}=0$, $\begin{vmatrix}d&e\\g&h\end{vmatrix}=0$, $\begin{vmatrix}a&c+a\\d&f+d\end{vmatrix}=0$, $\begin{vmatrix}a&c+a\\g&i+g\end{vmatrix}=0$,
$\begin{vmatrix}d&f+d\\g&i+g\end{vmatrix}=0$, $\begin{vmatrix}b&c+a\\e&f+d\end{vmatrix}=0$, $\begin{vmatrix}b&c+a\\h&i+g\end{vmatrix}=0$, $\begin{vmatrix}e&f+d\\h&i+g\end{vmatrix}=0$

veremos como esto implica que todos los menores de la matriz primitiva deban ser nulos.

- Los 3 de las dos primeras columnas son comunes.
- Los de la 1ª y 3ª columnas:

$$\begin{vmatrix}a&c+a\\d&f+d\end{vmatrix}=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}+\begin{vmatrix}a&a\\d&d\end{vmatrix}=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}+0=0 \Rightarrow \begin{vmatrix}a&c\\d&f\end{vmatrix}=0$$

Igual se haría con los otros dos.

- Los de la 2ª y 3ª columnas:

$$\left.\begin{array}{l}\begin{vmatrix}b&c+a\\e&f+d\end{vmatrix}=0\\ \begin{vmatrix}a&b\\d&e\end{vmatrix}=0\end{array}\right\} \Rightarrow 0=\begin{vmatrix}b&c\\e&f\end{vmatrix}+\begin{vmatrix}b&a\\e&d\end{vmatrix}=\begin{vmatrix}b&c\\e&f\end{vmatrix}-0=\begin{vmatrix}b&c\\e&f\end{vmatrix}$$

Igual se haría con los otros dos.

En definitiva, que la matriz primitiva no tendría rango 2.

por lo tanto $\boxed{\text{rango}\begin{pmatrix}a&b&c+a\\d&e&f+d\\g&h&i+g\end{pmatrix}=\text{rango}\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=2}$

**JUN 96**

**i)**

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{pmatrix}$$

$$B=\left(2^{i-j}\cdot a_{ij}\right)=\begin{pmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33}\end{pmatrix}$$

$$|B|=\begin{vmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ 2a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 4a_{31} & 2a_{32} & a_{33}\end{vmatrix}=2\begin{vmatrix} \frac{1}{2}a_{11} & \frac{1}{2}a_{12} & \frac{1}{4}a_{13} \\ a_{21} & a_{22} & \frac{1}{2}a_{23} \\ 2a_{31} & 2a_{32} & a_{33}\end{vmatrix}=\begin{vmatrix} \frac{1}{2}a_{11} & \frac{1}{2}a_{12} & \frac{1}{2}a_{13} \\ a_{21} & a_{22} & a_{23} \\ 2a_{31} & 2a_{32} & 2a_{33}\end{vmatrix}=$$

$$=2\cdot\frac{1}{2}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33}\end{vmatrix}=|A|$$

**ii)**

$$A=(a_{ij})=(i+j)=\begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6\end{pmatrix}$$

$$|A|=\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6\end{vmatrix}=0 \Rightarrow \boxed{\nexists\ A^{-1}}$$

**SEPT 96**

$$\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x\end{vmatrix}=\begin{vmatrix} x & 1 & 1 & 1 \\ 1-x & x-1 & 0 & 0 \\ 1-x & 0 & x-1 & 0 \\ 1-x^2 & 1-x & 1-x & 0\end{vmatrix}=-1\cdot\begin{vmatrix} 1-x & x-1 & 0 \\ 1-x & 0 & x-1 \\ 1-x^2 & 1-x & 1-x\end{vmatrix}=-\begin{vmatrix} -(x-1) & x-1 & 0 \\ -(x-1) & 0 & x-1 \\ -(x+1)(x-1) & -(x-1) & -(x-1)\end{vmatrix}=$$

$$\boxed{\begin{matrix} F2\to F2-F1 \\ F3\to F3-F1 \\ F4\to F4-xF1\end{matrix}}$$

$$=-(x-1)^3\begin{vmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -(x+1) & -1 & -1\end{vmatrix}=(x-1)^3\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ x+1 & -1 & -1\end{vmatrix}=(x-1)^3\begin{vmatrix} 1 & 1 & 0 \\ \cancel{1} & \cancel{0} & 1 \\ x+2 & -1 & 0\end{vmatrix}=-(x-1)^3\begin{vmatrix} 1 & 1 \\ x+2 & -1\end{vmatrix}=$$

$$\boxed{F3\to F3+F2}$$

$$=-(x-1)^3\begin{vmatrix} \cancel{1} & 1 \\ x+3 & 0\end{vmatrix}=+(x-1)^3(x+3)=(x-1)^3(x+3)$$

$$\boxed{F2\to F2+F1}$$

$$(x-1)^3(x+3)=0 \Rightarrow \boxed{\begin{matrix} x=+1 \text{ (raíz triple)} \\ x=-3\end{matrix}}$$

**SEPT 96**

$$M=\begin{pmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & a^2\end{pmatrix}$$

$$\begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a\end{vmatrix}=a^3+1+1-a-a-a=a^3-3a+2$$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ 1 & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \\ 1 & & 1 & 2 & \\ \hline & 1 & 2 & \boxed{0} & \\ -2 & & -2 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

- Si $\left.\begin{matrix} a\neq 1 \\ a\neq -2\end{matrix}\right| \Rightarrow \boxed{\text{rango } M=3}$

- Si $a=1 \Rightarrow M=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix} \rightarrow \boxed{\text{rango } M=1}$

- Si $a=-2 \Rightarrow M=\begin{pmatrix}-2&1&1&1\\1&-2&1&-2\\1&1&-2&4\end{pmatrix}$

$\begin{vmatrix}-2&1\\1&-2\end{vmatrix}=4-1=3 \Rightarrow$ rango $M\geq 2$

$\begin{vmatrix}-2&1&1\\1&-2&1\\1&1&-2\end{vmatrix}=0$

$\begin{vmatrix}-2&1&1\\1&-2&-2\\1&1&4\end{vmatrix}=16+1-2+2-4-4=9 \Rightarrow \boxed{\text{rango } M=3}$

Resumen: El rango es 3 para cualquier valor de a salvo para $a=1$ en el que el rango es 1.

JUN 97

i) $\begin{pmatrix}1&7&8\\3&1&K\end{pmatrix}=P\cdot\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}$

$(2\times3) = (2\times2)\cdot(2\times3)$

P tendría que ser una matriz $(2\times2)$

$\begin{pmatrix}1&7&8\\3&1&K\end{pmatrix}=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}=\begin{pmatrix}a+2b&2a-b&3a+b\\c+2d&2c-d&3c+d\end{pmatrix}$

$\left.\begin{array}{l}a+2b=1\\2a-b=7\\3a+b=8\end{array}\right\}$ $\begin{vmatrix}1&2&1\\2&-1&7\\3&1&8\end{vmatrix}=0 \Rightarrow$ rango $A^+=2$

$a=\dfrac{\begin{vmatrix}1&2\\7&-1\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\dfrac{-1-14}{-1-4}=\dfrac{-15}{-5}=3$

$b=\dfrac{\begin{vmatrix}1&1\\2&7\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\dfrac{5}{-5}=-1$

$\left.\begin{array}{l}c+2d=3\\2c-d=1\\3c+d=K\end{array}\right\} \rightarrow$ $c=\dfrac{\begin{vmatrix}3&2\\1&-1\end{vmatrix}}{\begin{vmatrix}1&2\\2&-1\end{vmatrix}}=\dfrac{-5}{-5}=1$, $d=\dfrac{\begin{vmatrix}1&3\\2&1\end{vmatrix}}{-5}=\dfrac{-5}{-5}=1$ $\Rightarrow K=3c+d=3\cdot1+1=4$

$\boxed{K \text{ debe ser } 4, \text{ y la matriz } P=\begin{pmatrix}3&-1\\1&1\end{pmatrix}}$

ii) $M\cdot N^t=\begin{pmatrix}1&2&3\\2&-1&1\end{pmatrix}\begin{pmatrix}1&3\\7&1\\8&K\end{pmatrix}=\begin{pmatrix}39&3K+5\\3&K+5\end{pmatrix}$

Tiene sentido hablar de la inversa de $M\cdot N^t$ puesto que es cuadrada, pero podría no existir si su determinante fuese nulo:

$\begin{vmatrix}39&3K+5\\3&K+5\end{vmatrix}=39K+195-9K-15=30K+180$

$30K+180=0 \Rightarrow \boxed{K=-6}$

Para $K\neq-6$, existe $(M\cdot N^t)^{-1}$.

Para $K=0 \Rightarrow M\cdot N^t=\begin{pmatrix}39&5\\3&5\end{pmatrix} \rightarrow \boxed{(MN^t)^{-1}=\begin{pmatrix}\frac{5}{180}&-\frac{5}{180}\\-\frac{3}{180}&\frac{39}{180}\end{pmatrix}}$

SEPT 97

i) $A^2 = I$, la única solución no tiene porqué ser $I$.

por ejemplo: $A = -I$, $(-I)\cdot(-I) = +I$ ✓

NOTA: para matrices $2\times 2$ se puede estudiar todas sus soluciones:

$A^2 = I \Rightarrow |A^2| = |I| \Rightarrow |A||A| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

$\Rightarrow A = A^{-1}$

Caso $|A| = 1$:

$$\left.\begin{matrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \\ |A| = 1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ |A| = 1\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a = d \\ b = -b \\ c = -c \\ d = a \\ ad - bc = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \boxed{\begin{matrix} b = 0 \\ c = 0 \end{matrix}} \\ a = d \\ ad = 1 \end{matrix}\right\} \rightarrow a^2 = 1 \Rightarrow \boxed{a = \begin{matrix} 1 \Rightarrow d = 1 \\ -1 \Rightarrow d = -1 \end{matrix}}$$

Hay dos soluciones: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Caso $|A| = -1$:

$$\left.\begin{matrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \\ |A| = -1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -d & b \\ c & -a \end{pmatrix} \\ |A| = 1\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a = -d \\ b = b \\ c = c \\ d = -a \\ ad - bc = -1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = -d \\ ad - bc = -1 \\ \boxed{\begin{matrix} b \in \mathbb{R} \\ c \in \mathbb{R} \end{matrix}} \end{matrix}\right\} \rightarrow \begin{matrix} -d^2 - bc = -1 \\ d^2 + bc = 1 \\ d = \boxed{\pm\sqrt{1-bc}} \rightarrow a = \boxed{\mp\sqrt{1-bc}} \end{matrix}$$

Hay infinitas soluciones del tipo: $\begin{pmatrix} \sqrt{1-bc} & b \\ c & -\sqrt{1-bc} \end{pmatrix}$, $\begin{pmatrix} -\sqrt{1-bc} & b \\ c & \sqrt{1-bc} \end{pmatrix}$

ii) $A^3 = I \Rightarrow \left.\begin{matrix} A\cdot A^2 = I \\ A^2\cdot A = I \end{matrix}\right\} \Rightarrow \boxed{A^{-1} = A^2}$

iii) $\left.\begin{matrix} (1)\ AB = A \\ (2)\ BA = B \end{matrix}\right\} \Rightarrow A^2 = (AB)\cdot(AB) = A\underbrace{(BA)}_{(2)}B = \overbrace{(AB)B}^{(1)} = \underbrace{AB}_{(1)} = A$

JUN 98

$$X\cdot\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

$(3\times 2)\cdot(2\times 2) = (3\times 2)$ $X$ es $(3\times 2)$

$$X = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}\cdot\begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix} = \boxed{\begin{pmatrix} 5 & -3 \\ 9 & -5 \\ 13 & -7 \end{pmatrix}}$$

La solución es única porque $\begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$ tiene inversa.

**SEPT 98**

i] M triangular superior $\Longleftrightarrow$ $a_{ij}=0 \quad \forall i>j$

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ 0 & a_{22} & \cdots & a_{2m} \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & a_{mm} \end{vmatrix} = a_{11}\,a_{22}\cdots a_{mm}$$

ii] $M=\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix}$

$$M^2=I \Rightarrow \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22}\end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix} \rightarrow \left.\begin{array}{l} a_{11}^2=1 \\ a_{11}a_{12}+a_{12}a_{22}=0 \\ 0=0 \\ a_{22}^2=1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a_{11}=\pm1 \\ a_{22}=\pm1 \\ a_{12}\cdot(a_{11}+a_{22})=0 \end{array}\right\}$$

- $\boxed{\begin{array}{l} a_{11}=1 \\ a_{22}=1\end{array}}$

  $2a_{12}=0 \rightarrow \boxed{a_{12}=0}$

- $\boxed{\begin{array}{l} a_{11}=1 \\ a_{22}=-1\end{array}}$

  $a_{12}\cdot 0=0 \rightarrow \boxed{a_{12}\in\mathbb{R}}$

- $\boxed{\begin{array}{l} a_{11}=-1 \\ a_{22}=1\end{array}}$

  $a_{12}\cdot 0=0 \rightarrow \boxed{a_{12}\in\mathbb{R}}$

- $\boxed{\begin{array}{l} a_{11}=-1 \\ a_{22}=-1\end{array}}$

  $-2a_{12}=0 \rightarrow \boxed{a_{12}=0}$

$$\boxed{\begin{array}{c} M=\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix} \\[2em] M=\begin{pmatrix} 1 & a_{12} \\ 0 & -1\end{pmatrix} \\[2em] M=\begin{pmatrix} -1 & a_{12} \\ 0 & 1\end{pmatrix} \\[2em] M=\begin{pmatrix} -1 & 0 \\ 0 & -1\end{pmatrix}\end{array}}$$

**JUN 99**

$$(x \;\; y \;\; z)\begin{pmatrix} 1 & -2 \\ 2 & 1 \\ 1 & 2\end{pmatrix}=(0 \;\; 0) \rightarrow (x+2y+z \quad -2x+y+2z)=(0 \quad 0) \rightarrow$$

$$\rightarrow \left.\begin{array}{l} x+2y+z=0 \\ -2x+y+2z=0\end{array}\right\} \rightarrow \begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 2\end{pmatrix}\begin{pmatrix} x \\ y \\ z\end{pmatrix}=\begin{pmatrix} 0 \\ 0\end{pmatrix} \qquad \boxed{A=\begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 2\end{pmatrix}}$$

$$\left.\begin{array}{l} x+2y=-z \\ -2x+y=-2z\end{array}\right\} \qquad x=\frac{\begin{vmatrix} -z & 2 \\ -2z & 1\end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -2 & 1\end{vmatrix}}=\frac{-z+4z}{5}=\frac{3}{5}z$$

$$y=\frac{\begin{vmatrix} 1 & -z \\ -2 & -2z\end{vmatrix}}{5}=\frac{-2z-2z}{5}=-\frac{4}{5}z$$

$z\in\mathbb{R}$

$$\sqrt{x^2+y^2+z^2}=1 \Rightarrow \left(\frac{3}{5}z\right)^2+\left(-\frac{4}{5}z\right)^2+z^2=1 \Rightarrow \left(\frac{9}{25}+\frac{16}{25}+1\right)z^2=1 \Rightarrow$$

$$\Rightarrow z^2=\frac{1}{2} \Rightarrow z=\pm\frac{\sqrt{2}}{2}$$

Soluciones con módulo 1: $\boxed{\left(\frac{3\sqrt{2}}{10}, -\frac{4\sqrt{2}}{10}, \frac{\sqrt{2}}{2}\right) \qquad \left(-\frac{3\sqrt{2}}{10}, \frac{4\sqrt{2}}{10}, -\frac{\sqrt{2}}{2}\right)}$

JUN 99

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & 0 & b \end{vmatrix} = b$$

i) $|A| = \operatorname{sen}\alpha \Rightarrow b = \operatorname{sen}\alpha \Rightarrow \boxed{b \in [-1,1]}$

ii) Existe si $b \neq 0$:

$A_{11} = \begin{vmatrix} 1 & 0 \\ 0 & b \end{vmatrix} = b$ ; $A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & b \end{vmatrix} = 0$ ; $A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{12} = -\begin{vmatrix} 0 & 0 \\ a & b \end{vmatrix} = 0$ ; $A_{22} = \begin{vmatrix} 1 & 0 \\ a & b \end{vmatrix} = b$ ; $A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} 0 & 1 \\ a & 0 \end{vmatrix} = -a$ ; $A_{23} = -\begin{vmatrix} 1 & 0 \\ a & 0 \end{vmatrix} = 0$ ; $A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -a/b & 0 & 1/b \end{pmatrix}}$$ siendo $b \neq 0$

iii) $A = A^{-1} \Rightarrow \left.\begin{matrix} a = -\frac{a}{b} \\ b = \frac{1}{b} \end{matrix}\right\} \rightarrow \left.\begin{matrix} ab + a = 0 \\ b^2 = 1 \end{matrix}\right\} \rightarrow \left.\begin{matrix} a(b+1) = 0 \\ b^2 = 1 \end{matrix}\right\} \rightarrow b = \pm 1$

- $\boxed{b=1} \rightarrow 2a = 0 \Rightarrow \boxed{a = 0}$
- $\boxed{b=-1} \rightarrow a \cdot 0 = 0 \Rightarrow \boxed{a \in \mathbb{R}}$

SEPT 99

i) $$|A| = \begin{vmatrix} \lambda & 1 & 1 \\ -1 & 2 & \lambda \\ 1 & 1 & 3 \end{vmatrix} = 6\lambda - 1 + \lambda - 2 + 3 - \lambda^2 = -\lambda^2 + 7\lambda$$

$|A| = 0 \Rightarrow -\lambda^2 + 7\lambda = 0$ ; $\lambda = 0, 7$

Para $\lambda = 0$ o $\lambda = 7$ A no tiene inversa.

Para $\left.\begin{matrix} \lambda \neq 0 \\ \lambda \neq 7 \end{matrix}\right\}$ A tiene inversa.

ii) $|bA| = b^3|A| = b^3(7\lambda - \lambda^2) = 1 \Rightarrow \boxed{b = \sqrt[3]{\frac{1}{7\lambda - \lambda^2}}}$

JUN 00

i) $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ $|A| = ab$

$A = A^{-1} \Rightarrow \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{b} \end{pmatrix} \Rightarrow \left.\begin{matrix} a = 1/a \\ b = 1/b \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2 = 1 \\ b^2 = 1 \end{matrix}\right\} \Rightarrow \boxed{\begin{matrix} a = \pm 1 \\ b = \pm 1 \end{matrix}}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

ii) $A^2 = A \cdot A = A \cdot A^{-1} = \boxed{I}$

SEPT 00

i) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$|A| = |A + I| \Rightarrow \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a+1 & b \\ c & d+1 \end{vmatrix} \Rightarrow ad - bc = (a+1)(d+1) - bc \Rightarrow$

$\Rightarrow ad = ad + a + d + 1 \Rightarrow \boxed{a + d + 1 = 0} \rightarrow \boxed{\begin{matrix} a \in \mathbb{R} \\ d = -a-1 \\ b \in \mathbb{R} \\ c \in \mathbb{R} \end{matrix}}$

$$\boxed{A = \begin{pmatrix} a & b \\ c & -a-1 \end{pmatrix}}$$

**ii)** $|A| = |A+I|$, $b = c = 0$, $|A| = 0$ $\Rightarrow \begin{vmatrix} a & 0 \\ 0 & -a-1 \end{vmatrix} = 0 \Rightarrow a(-a-1) = 0 \Rightarrow a = 0,\ -1$

$\boxed{A = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}}$ $\boxed{A = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}}$

**iii)** Casi nunca se cumple. Por ejemplo:

$|I+I| = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$

$|I| + |I| = 1 + 1 = 2$

**JUN 01**

**a)** $B \cdot A$ = matriz fila

$(1\times m)(m\times m)$ = matriz fila

$\boxed{B \text{ debe ser } 1\times m}$

**b)** $A \cdot B$ = matriz fila

$(m\times m)\cdot(m\times\ )$ = matriz fila

$\boxed{\text{Tendría que ser } m = 1, \text{ de ser así, } B \text{ debería tener } m \text{ filas}}$

**c)** $B \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$(1\times 3)\cdot(3\times 2) = (1\times 2)$

$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$\boxed{a = 0}$, $2a + b = 0$ $\longrightarrow \boxed{b = 0}$

$\boxed{B = \begin{pmatrix} 0 & 0 & c \end{pmatrix},\ c \in \mathbb{R}}$

**SEPT 01**

$A \cdot B \cdot A = C$

Si $A$ es $m\times m$, $\boxed{B \text{ será } m\times m}$

$ABA = C \Rightarrow B = A^{-1} C A^{-1} = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} =$

$= \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -6 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} =$

$= \boxed{\begin{pmatrix} -1 & 2 \\ 0 & -3 \end{pmatrix}}$

**JUN 02**

**a)** $C(A+X)B = I$

$A + X = C^{-1} I B^{-1}$

$X = \boxed{C^{-1}B^{-1} - A}$

**b)** $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \rightarrow B^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \rightarrow C^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

$X = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & -5 \\ -2 & 0 \end{pmatrix}}$

SEPT 02

$$\begin{vmatrix} 2a & a & a & a \\ a & 2a & a & a \\ a & a & 2a & a \\ a & a & a & 2a \end{vmatrix} = a^4 \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = a^4 \begin{vmatrix} \cancel{2} & 1 & 1 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -3 & -1 & -1 & 0 \end{vmatrix} = -a^4 \begin{vmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -3 & -1 & -1 \end{vmatrix} =$$

$$\boxed{\begin{array}{l} F2 \to F2 - F1 \\ F3 \to F3 - F1 \\ F4 \to F4 - 2 \cdot F1 \end{array}}$$

$$= -a^4 (-3-1-1) = \boxed{5a^4}$$

$$M = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix} \qquad |M| = 5$$

$$m_{11} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4 \qquad m_{21} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{31} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{41} = -\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -1$$

$$m_{12} = -\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{22} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4 \qquad m_{32} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{42} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -1$$

$$m_{13} = +\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{23} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -1 \qquad m_{33} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4 \qquad m_{43} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1$$

$$m_{14} = -\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -1 \qquad m_{24} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -1 \qquad m_{34} = -\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -1 \qquad m_{44} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 4$$

$$\boxed{M^{-1} = \begin{pmatrix} 4/5 & -1/5 & -1/5 & -1/5 \\ -1/5 & 4/5 & -1/5 & -1/5 \\ -1/5 & -1/5 & 4/5 & -1/5 \\ -1/5 & -1/5 & -1/5 & 4/5 \end{pmatrix}}$$

SEPT 02

a) $AC = I \Rightarrow C = A^{-1} = \begin{pmatrix} -1/7 & 3/7 \\ 3/7 & -2/7 \end{pmatrix}$

$BD = I \Rightarrow D = B^{-1} = \begin{pmatrix} 1 & 0 \\ 1/5 & -1/5 \end{pmatrix}$

b) $C^{-1} = (A^{-1})^{-1} = A = \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$

$D^{-1} = (B^{-1})^{-1} = B = \begin{pmatrix} 1 & 0 \\ 1 & -5 \end{pmatrix}$

$$\left[ \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & -5 \end{pmatrix} \right] \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longrightarrow \left. \begin{array}{l} x + 3y = 1 \\ 2x + 6y = 2 \end{array} \right\}$$

$$\left. \begin{array}{lll} M = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} & |M| = 0 \Rightarrow & \text{rango } M = 1 \\ M^+ = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{pmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0 \Rightarrow & \text{rango } M^+ = 1 \\ & & n = 2 \end{array} \right\} \Rightarrow \boxed{\text{S. Compatible Indeterminado}}$$

$$\boxed{\begin{array}{l} x = 1 - 3y \\ y \in \mathbb{R} \end{array}}$$ Son las infinitas soluciones.

JUN 03

a) $\left.\begin{array}{l}(B-C)\cdot A=0\\ A \text{ no singular}\end{array}\right\} \Rightarrow (B-C)=0\cdot A^{-1} \Rightarrow B-C=0 \Rightarrow \boxed{B=C}$

b) $\left.\begin{array}{l}XA=0\\ A \text{ no singular}\end{array}\right\} \Rightarrow \boxed{X=0}$ Sería la única solución, siempre que A sea no singular.

Si A es singular, no se podría deducir como única solución $X=0$.

$|A| = \begin{vmatrix} 2 & -6 \\ -1 & 3 \end{vmatrix} = 0$ . A es singular.

$$\begin{pmatrix} & \\ & \end{pmatrix} \cdot \begin{pmatrix} 2 & -6 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Habría infinitas soluciones para X. Por ejemplo: $X=\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $X=\begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$, ⋮

SEPT 03

a) $|A| = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1 \rightarrow$ Existe $A^{-1}$

$A_{11} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$, $A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$, $A_{13} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$A_{21} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$, $A_{22} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$, $A_{23} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{31} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$, $A_{32} = -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$, $A_{33} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$$\boxed{A^{-1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}$$

b) $A^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$A^3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

$A^4 = A^3 \cdot A = I \cdot A = A$

$A^5 = A^4 \cdot A = A \cdot A = A^2$

Es decir:

$A = A^4 = A^7 = \cdots\cdots = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} = A^n$ (con $n=\dot{3}+1$)

$A^2 = A^5 = A^8 = \cdots\cdots = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = A^n$ (con $n=\dot{3}+2$)

$A^3 = A^6 = A^9 = \cdots\cdots = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = A^n$ (con $n=\dot{3}$)

**JUN 04**

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ -2 & 1 & x \\ 1 & x & 0 \end{vmatrix} = -4x - 2 - x^2$$

**a)**

$$-x^2 - 4x - 2 = 0$$

$$x = \frac{4 \pm \sqrt{16-8}}{-2} = \frac{4 \pm 2\sqrt{2}}{-2} \begin{cases} -2+\sqrt{2} \\ -2-\sqrt{2} \end{cases}$$

$\boxed{\text{Para } x \neq -2 \pm \sqrt{2} \quad A \text{ tendrá inversa}}$

**b)**

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \qquad |A| = 1$$

$A_{11} = \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -1$ ; $A_{21} = -\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} = -2$ ; $A_{31} = \begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = -2$

$A_{12} = -\begin{vmatrix} -2 & -1 \\ 1 & 0 \end{vmatrix} = -1$ ; $A_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2$ ; $A_{32} = -\begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} = -3$

$A_{13} = \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} = 1$ ; $A_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = +1$ ; $A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix}}$$

**c)**

$$A \cdot B = C \cdot D$$

$$(3\times3)\cdot(3\times2) = (3\times2)\cdot(2\times2)$$

$\boxed{B \text{ debe ser } 3\times2}$

$$B = A^{-1}CD = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -2 \\ -1 & -2 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & -2 \\ -1 & -2 \\ 1 & 1 \end{pmatrix}}$$

**SEPT 04**

$$|A| = \begin{vmatrix} m & 2 & 6 \\ 2 & m & 4 \\ 2 & m & 6 \end{vmatrix} = 6m^2 + 12m + 16 - 12m - 24 - 4m^2 = 2m^2 - 8$$

**a)**

$2m^2 - 8 = 0$ ; $m^2 = 4$ ; $\boxed{m = \pm 2}$

- Si $\left.\begin{matrix} m \neq 2 \\ m \neq -2 \end{matrix}\right\} \Rightarrow \boxed{\text{rango } A = 3}$
- Si $m = 2 \longrightarrow A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 2 & 4 \\ 2 & 2 & 6 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 6 \\ 2 & 4 \end{vmatrix} = 8 - 12 = -4 \Rightarrow \boxed{\text{rango } A = 2}$
- Si $m = -2 \longrightarrow A = \begin{pmatrix} -2 & 2 & 6 \\ 2 & -2 & 4 \\ 2 & -2 & 6 \end{pmatrix}$ $\quad \begin{vmatrix} 2 & 6 \\ -2 & 4 \end{vmatrix} = 8 + 12 = 20 \Rightarrow \boxed{\text{rango } A = 2}$

**b)**

$$A \cdot X = B$$

$$(3\times3)(3\times2) = (3\times2)$$

$\boxed{X \text{ debe ser } 3\times2}$

**c)**

$$A \cdot X = B \Rightarrow X = A^{-1}B$$

$\underline{m=0}$: $A = \begin{pmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 2 & 0 & 6 \end{pmatrix}$ $\quad |A| = -8$

$A_{11} = \begin{vmatrix} 0 & 4 \\ 0 & 6 \end{vmatrix} = 0$ ; $A_{21} = -\begin{vmatrix} 2 & 6 \\ 0 & 6 \end{vmatrix} = -12$ ; $A_{31} = \begin{vmatrix} 2 & 6 \\ 0 & 4 \end{vmatrix} = 8$

$A_{12} = -\begin{vmatrix} 2 & 4 \\ 2 & 6 \end{vmatrix} = -4$ ; $A_{22} = \begin{vmatrix} 0 & 6 \\ 2 & 6 \end{vmatrix} = -12$ ; $A_{32} = \begin{vmatrix} 0 & 6 \\ 2 & 4 \end{vmatrix} = -12$

$A_{13} = \begin{vmatrix} 2 & 0 \\ 2 & 0 \end{vmatrix} = 0$ ; $A_{23} = -\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = 4$ ; $A_{33} = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4$

$$\boxed{A^{-1} = \begin{pmatrix} 0 & 3/2 & -1 \\ 1/2 & 3/2 & -3/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} 0 & 3/2 & -1 \\ 1/2 & 3/2 & -3/2 \\ 0 & -1/2 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 2 \\ 1 & 0 \\ -1 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 5/2 & -2 \\ 4 & -2 \\ -1 & 1 \end{pmatrix}}$$

**JUN 05**

$$\begin{vmatrix} 0 & 1 & x \\ x & x & 1 \\ -x & 1 & x \end{vmatrix} = x^2 - x + x^3 - \cancel{x^2} = x^3 - x$$

$$x^3 - x = 0 \quad ; \quad +x(x^2 - 1) = 0 \implies \boxed{\begin{array}{l} x = 0 \\ x = 1 \\ x = -1 \end{array}}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x^2 \end{vmatrix} = x^3 + 1 + 1 - x - x^2 - \cancel{1} = x^3 - x^2 - x + 1$$

$$\begin{array}{r|rrrr} & 1 & -1 & -1 & 1 \\ 1 & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & \boxed{0} \\ 1 & & 1 & 1 & \\ \hline & 1 & 1 & \boxed{0} & \\ -1 & & -1 & & \\ \hline & 1 & \boxed{0} & & \end{array}$$

$$\boxed{\begin{array}{l} x = 1 \text{ (raíz doble)} \\ x = -1 \end{array}}$$

**SEPT 05**

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = K$$

$$\begin{vmatrix} d & 2e & f \\ a & 2b & c \\ g & 2h & i \end{vmatrix} = 2 \begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \boxed{-2K}$$

2

$$\begin{vmatrix} a+b & b & 2c \\ d+e & e & 2f \\ g+h & h & 2i \end{vmatrix} = \begin{vmatrix} a & b & 2c \\ d & e & 2f \\ g & h & 2i \end{vmatrix} + \begin{vmatrix} b & b & 2c \\ e & e & 2f \\ h & h & 2i \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + 0 = \boxed{2K}$$

2 iguales

**JUN 06**

a) $|A| = \begin{vmatrix} 1 & 0 & -1 \\ 0 & x & 3 \\ 4 & 1 & -x \end{vmatrix} = -x^2 + 4x - 3$

$-x^2 + 4x - 3 = 0 \longrightarrow x = \begin{cases} 1 \\ 3 \end{cases}$

$\boxed{A \text{ tiene inversa } \forall x \in \mathbb{R} - \{1, 3\}}$

b) $x = 2 \longrightarrow A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \\ 4 & 1 & -2 \end{pmatrix}$

$|A| = -4 + 8 - 3 = 1$

$A_{11} = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7 \qquad A_{21} = -\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = -1 \qquad A_{31} = \begin{vmatrix} 0 & -1 \\ 2 & 3 \end{vmatrix} = 2$

$A_{12} = -\begin{vmatrix} 0 & 3 \\ 4 & -2 \end{vmatrix} = 12 \qquad A_{22} = \begin{vmatrix} 1 & -1 \\ 4 & -2 \end{vmatrix} = 2 \qquad A_{32} = -\begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = -3$

$A_{13} = \begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix} = -8 \qquad A_{23} = -\begin{vmatrix} 1 & 0 \\ 4 & 1 \end{vmatrix} = -1 \qquad A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2$

$$\boxed{A^{-1} = \begin{pmatrix} -7 & -1 & 2 \\ 12 & 2 & -3 \\ -8 & -1 & 2 \end{pmatrix}}$$

c) $x = 5 \longrightarrow A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 5 & 3 \\ 4 & 1 & -5 \end{pmatrix} \longrightarrow |A| = -25 + 20 - 3 = -8$

$|bA| = b^3 \cdot |A| = -8b^3$

$-8b^3 = 1 \implies b^3 = \dfrac{-1}{8} \qquad \boxed{b = -\dfrac{1}{2}}$

**SEPT 06**

a) $B\cdot A = \begin{pmatrix} k & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & k \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} k & -1 \\ 3 & k+2 \end{pmatrix}$

$|B\cdot A| = k^2+2k+3$

$k^2+2k+3=0 \longrightarrow k = \dfrac{-2\pm\sqrt{4-12}}{2} = \dfrac{-2\pm 2i\sqrt{2}}{2} = -1\pm i\sqrt{2}$

$\boxed{\text{Entonces } B\cdot A \text{ tiene inversa } \forall x\in\mathbb{R}}$

b) $A\cdot B = \begin{pmatrix} 1 & 0 \\ 2 & k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} k & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} k & 0 & -1 \\ 3k & k & 2k-2 \\ 1 & 1 & 2 \end{pmatrix}$

$|A\cdot B| = 2k^2-3k+k-2k^2+2k = 0$

$\boxed{\text{Entonces } A\cdot B \text{ no tiene inversa para ningún } x\in\mathbb{R}}$

**JUN 07**

a) $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & a & 1 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \longrightarrow r(A)\geq 2$

$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & a & 1 \end{vmatrix} = 2a+2-1 = 2a+1$

$2a+1=0 \longrightarrow a=-1/2$

$\boxed{\begin{array}{l} \text{Si } a=-1/2 \Rightarrow r(A)=2 \\ \text{Si } a\neq -1/2 \Rightarrow r(A)=3 \end{array}}$

$B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & a & 1 & 1+a \end{pmatrix}$

Para $a\neq -1/2 \Rightarrow r(B)=3$ (Hecho con A)

Para $a=-1/2 \Rightarrow B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1/2 & 1 & 1/2 \end{pmatrix}$

$\begin{vmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & -1/2 & 1/2 \end{vmatrix} = -\dfrac{3}{2}+2-\dfrac{1}{2} = 0$

$\boxed{\begin{array}{l} \text{Si } a=-1/2 \Rightarrow r(B)=2 \\ \text{Si } a\neq -1/2 \Rightarrow r(B)=3 \end{array}}$

b) $a=-1 \longrightarrow A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix}$

$A\cdot X = B \Rightarrow \boxed{X = A^{-1}\cdot B}$ (Como $a=-1\neq -1/2 \Rightarrow \exists A^{-1}$)

$|A| = -2+2-1 = -1$

$A_{11} = \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 2 \quad A_{12} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 \quad A_{13} = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1$

$A_{21} = -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -3 \quad A_{22} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2 \quad A_{23} = -\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} = 1$

$A_{31} = \begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2 \quad A_{32} = -\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} = 2 \quad A_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$\Rightarrow A^{-1} = \begin{pmatrix} -2 & 3 & -2 \\ -1 & 2 & -2 \\ 1 & -1 & 1 \end{pmatrix}$

$X = \begin{pmatrix} -2 & 3 & -2 \\ -1 & 2 & -2 \\ 1 & -1 & 1 \end{pmatrix}\cdot\begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 2 \\ 1 & -1 & 1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}}$

**SEPT 07**

$$A=\begin{pmatrix}-1&-2&-2\\1&2&1\\0&-1&-1\end{pmatrix}$$

**a)**

$$A^2=A\cdot A=\begin{pmatrix}-1&0&2\\1&1&-1\\-1&-1&0\end{pmatrix}$$

$$A^3=A^2\cdot A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I \Rightarrow \boxed{A^3-I=0}$$

**b)**

$$A^{13}=A^3\cdot A^3\cdot A^3\cdot A^3\cdot A = A = \boxed{\begin{pmatrix}-1&-2&-2\\1&2&1\\0&-1&-1\end{pmatrix}}$$

NOTA: $\begin{array}{r|l}13 & 3\\ \hline 1 & 4\end{array} \Rightarrow A^{13}=A^{12}\cdot A=(A^3)^4\cdot A$

**c)**

$$A^2X+I=A \longrightarrow A\cdot A^2X+A\cdot I=A\cdot A \longrightarrow A^3X+A=A^2 \longrightarrow IX+A=A^2 \Rightarrow$$

$$\Rightarrow X=A^2-A=\begin{pmatrix}-1&0&2\\1&1&-1\\-1&-1&0\end{pmatrix}-\begin{pmatrix}-1&-2&-2\\1&2&1\\0&-1&-1\end{pmatrix}=\boxed{\begin{pmatrix}0&2&4\\0&-1&-2\\-1&0&1\end{pmatrix}}$$

**JUN 08**

$$A=\begin{pmatrix}x&y&x\\y&0&y\\1&z&z\end{pmatrix} \qquad B=(a\ \ 2\ \ 3) \qquad C=(4,0,2)$$

**a)**

$$|A|=\begin{vmatrix}x&y&x\\y&0&y\\1&z&z\end{vmatrix}=\cancel{xyz}+y^2-y^2z-\cancel{xyz}=y^2(1-z)$$

Para $\left.\begin{array}{l}x\in\mathbb{R}\\ y\in\mathbb{R}-\{0\}\\ z\in\mathbb{R}-\{1\}\end{array}\right\}$ existe $A^{-1}$  $\boxed{\text{Para } y=0 \text{ ó } z=1 \ \nexists A^{-1}}$

**b)**

$B\cdot A=C$

$$(a\ \ 2\ \ 3)\begin{pmatrix}x&y&x\\y&0&y\\1&z&z\end{pmatrix}=(4\ \ 0\ \ 2)$$

$$\left.\begin{array}{r}ax+2y+3=4\\ ay+3z=0\\ ax+2y+3z=2\end{array}\right\} \longrightarrow \left.\begin{array}{r}ax+2y=1\\ ay+3z=0\\ ax+2y+3z=2\end{array}\right\}$$

$$M=\begin{pmatrix}a&2&0\\0&a&3\\a&2&3\end{pmatrix} \qquad \begin{vmatrix}2&0\\a&3\end{vmatrix}=6\neq0 \Rightarrow r(M)\geq 2$$

$$|M|=3a^2+\cancel{6a}-\cancel{6a}=3a^2$$

- $\boxed{\text{Si } a\neq 0} \Rightarrow r(M)=3 \Rightarrow$ $\boxed{\text{El sistema tiene solución única}}$
- $\boxed{\text{Si } a=0} \Rightarrow M=\begin{pmatrix}0&2&0\\0&0&3\\0&2&3\end{pmatrix}$  $r(M)=2$

$$M^*=\begin{pmatrix}0&2&0&1\\0&0&3&0\\0&2&3&2\end{pmatrix} \qquad \begin{vmatrix}2&0&1\\0&3&0\\2&3&2\end{vmatrix}=12-6=6\neq0 \Rightarrow r(M^*)=3 \Bigg\} \Rightarrow$$

$\Rightarrow$ $\boxed{\text{El sistema no tiene solución}}$

**c)** Para $a\neq 0$:

$$x=\frac{\begin{vmatrix}1&2&0\\0&a&3\\2&2&3\end{vmatrix}}{3a^2}=\frac{3a+12-6}{3a^2}=\frac{3a+6}{3a^2}=\boxed{\frac{a+2}{a}}$$

$$y=\frac{\begin{vmatrix}a&1&0\\0&0&3\\a&2&3\end{vmatrix}}{3a^2}=\frac{3a-6a}{3a^2}=\frac{-3a}{3a^2}=\boxed{-\frac{1}{a}}$$

$$z=\frac{\begin{vmatrix}a&2&1\\0&a&0\\a&2&2\end{vmatrix}}{3a^2}=\frac{2a^2-a^2}{3a^2}=\frac{a^2}{3a^2}=\boxed{\frac{1}{3}}$$

SEPT 08

$$A^2 = 6A - 9I$$

a) $A^4 = A^2 \cdot A^2 = (6A-9I)\cdot(6A-9I) = 36A^2 - 54A - 54A - 81I =$

$= 36(6A-9I) - 108A - 81I = 216A - 324I - 108A - 81I =$

$= \boxed{108A - 405I}$

b) $B = \begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix}$

$B^2 = \begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 18 & 6 \\ -12 & 27 & 6 \\ 12 & -18 & 3 \end{pmatrix}$ ✓

$6B - 9I = \begin{pmatrix} 6 & 18 & 6 \\ -12 & 36 & 6 \\ 12 & -18 & 12 \end{pmatrix} - \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} -3 & 18 & 6 \\ -12 & 27 & 6 \\ 12 & -18 & 3 \end{pmatrix}$ ✓

$\boxed{B^2 = 6B - 9I}$

c) $|B| = \begin{vmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ 2 & -3 & 2 \end{vmatrix} = 12 + 6 + 6 - \cancel{12} + \cancel{12} + 3 = 27$

$B_{11} = \begin{vmatrix} 6 & 1 \\ -3 & 2 \end{vmatrix} = 15$ $\quad B_{21} = -\begin{vmatrix} 3 & 1 \\ -3 & 2 \end{vmatrix} = -9$ $\quad B_{31} = \begin{vmatrix} 3 & 1 \\ 6 & 1 \end{vmatrix} = -3$

$B_{12} = -\begin{vmatrix} -2 & 1 \\ 2 & 2 \end{vmatrix} = 6$ $\quad B_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$ $\quad B_{32} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3$

$B_{13} = \begin{vmatrix} -2 & 6 \\ 2 & -3 \end{vmatrix} = -6$ $\quad B_{23} = -\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = 9$ $\quad B_{33} = \begin{vmatrix} 1 & 3 \\ -2 & 6 \end{vmatrix} = 12$

$$B^{-1} = \begin{pmatrix} 15/27 & -9/27 & -3/27 \\ 6/27 & 0 & -3/27 \\ -6/27 & 9/27 & 12/27 \end{pmatrix} = \boxed{\begin{pmatrix} 5/9 & -1/3 & -1/9 \\ 2/9 & 0 & -1/9 \\ -2/9 & 1/3 & 4/9 \end{pmatrix}}$$

JUN 09

a) $P = \begin{pmatrix} 0 & 1 & 3 \\ 2-a & 1 & a \\ 3 & 3 & a \end{pmatrix}$ $\qquad \begin{vmatrix} 0 & 3 \\ 3 & a \end{vmatrix} = -9 \neq 0 \Rightarrow r(P) \geq 2$

$|P| = \begin{vmatrix} 0 & 1 & 3 \\ 2-a & 1 & a \\ 3 & 3 & a \end{vmatrix} = 18 - 9a + 3a - 9 - 2a + a^2 = a^2 - 8a + 9$

$a^2 - 8a - 9 = 0$

$a = \dfrac{8 \pm \sqrt{64+36}}{2} = \dfrac{8 \pm 10}{2}$ ↗ $9$, ↘ $-1$

- Si $a = 9 \longrightarrow$ rango $(P) = 2$
- Si $a = -1 \longrightarrow$ rango $(P) = 2$
- Si $a \neq 9$, $a \neq -1 \longrightarrow$ rango $(P) = 3$

b) $a = 1$: $\quad PX = Q$

Como existe $P^{-1}$, $\quad \boxed{X = P^{-1} \cdot Q}$

Veamos la inversa de $P$: $\qquad P = \begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 3 & 1 \end{pmatrix}$

$|P| = 1^2 - 8 \cdot 1 + 9 = 2$

$$P_{11} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = -2 \qquad P_{21} = -\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 8 \qquad P_{31} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -2$$

$$P_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2 \qquad P_{22} = \begin{vmatrix} 0 & 3 \\ 3 & 1 \end{vmatrix} = -9 \qquad P_{32} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = 3$$

$$P_{13} = \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 0 \qquad P_{23} = -\begin{vmatrix} 0 & 1 \\ 3 & 3 \end{vmatrix} = 3 \qquad P_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$$

$$P^{-1} = \frac{1}{2} \cdot \begin{pmatrix} -2 & 8 & -2 \\ 2 & -9 & 3 \\ 0 & 3 & -1 \end{pmatrix}$$

$$X = P^{-1}Q = \frac{1}{2}\begin{pmatrix} -2 & 8 & -2 \\ 2 & -9 & 3 \\ 0 & 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -4 & 14 & -14 \\ 5 & -15 & 17 \\ -1 & 5 & -5 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & 7 & -7 \\ 5/2 & -15/2 & 17/2 \\ -1/2 & 5/2 & -5/2 \end{pmatrix}}$$

SEPT 09

a) $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & m & 1 \\ m & 1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 1 \\ m & 1 & 1 \end{vmatrix} = m + 1 + m - m^2 - 1 - 1 = -m^2 + 2m - 1$$

$-m^2 + 2m - 1 = 0$ ; $m = 1$

$\boxed{\text{Si } m \neq 1 \text{ existe } A^{-1}}$

b) $m = 2$ $\qquad A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$

$|A| = -4 + 4 - 1 = -1$

$$A_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \qquad A_{21} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \qquad A_{31} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 \qquad A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 \qquad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A_{13} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \qquad A_{23} = -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 \qquad A_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

$$\boxed{A^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}}$$

c) $m = 2$ , como $\exists A^{-1} \Rightarrow$ $X = A^{-1} \cdot B$

$$X = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & -1 & -1 \end{pmatrix}\begin{pmatrix} -4 \\ 1 \\ 4 \end{pmatrix} = \boxed{\begin{pmatrix} 8 \\ 5 \\ -17 \end{pmatrix}}$$

JUNIO fase General

a) $$A - mI = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} - m\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{pmatrix}$$

$$|A - mI| = \begin{vmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{vmatrix} = (1-m)^3 - 4(1-m) = (1-m)\left[(1-m)^2 - 4\right] = (1-m)(m^2 - 2m - 3)$$

$|A - mI| = 0 \longrightarrow (1-m)(m^2 - 2m - 3) = 0$
- $\to 1 - m = 0 \Rightarrow m = 1$
- $\to m^2 - 2m - 3 = 0 \Rightarrow m = -1,\ 3$

$\boxed{\text{Para } m=1,\ m=-1,\ m=3 \text{ , } A - mI \text{ no tiene inversa}}$

b) $B = A - mI = \begin{pmatrix} 1-m & 2 & 0 \\ 2 & 1-m & 1 \\ 0 & 0 & 1-m \end{pmatrix}$

$|B| = (1-m)(m^2 - 2m - 3)$

$$B_{11} = \begin{vmatrix} 1-m & 1 \\ 0 & 1-m \end{vmatrix} = (1-m)^2 \qquad B_{21} = -\begin{vmatrix} 2 & 0 \\ 0 & 1-m \end{vmatrix} = -2(1-m) \qquad B_{31} = \begin{vmatrix} 2 & 0 \\ 1-m & 1 \end{vmatrix} = 2$$

$$B_{12} = -\begin{vmatrix} 2 & 1 \\ 0 & 1-m \end{vmatrix} = -2(1-m) \qquad B_{22} = \begin{vmatrix} 1-m & 0 \\ 0 & 1-m \end{vmatrix} = (1-m)^2 \qquad B_{32} = -\begin{vmatrix} 1-m & 0 \\ 2 & 1 \end{vmatrix} = -(1-m)$$

$$B_{13} = \begin{vmatrix} 2 & 1-m \\ 0 & 0 \end{vmatrix} = 0 \qquad B_{23} = -\begin{vmatrix} 1-m & 2 \\ 0 & 0 \end{vmatrix} = 0 \qquad B_{33} = \begin{vmatrix} 1-m & 2 \\ 2 & 1-m \end{vmatrix} = (1-m)^2 - 4 = m^2 - 2m - 3$$

$$\boxed{(A - mI)^{-1} = \frac{1}{(1-m)(m^2 - 2m - 3)} \cdot \begin{pmatrix} (1-m)^2 & -2(1-m) & 2 \\ -2(1-m) & (1-m)^2 & -(1-m) \\ 0 & 0 & m^2 - 2m - 3 \end{pmatrix}}$$

JUN10 fase específica

a) $|A| = \begin{vmatrix} -x & 1 & 1 \\ 1 & -x & 1 \\ 1 & 1 & -x \end{vmatrix} = -x^3+1+1+x+x+x = -x^3+3x+2$

$-x^3+3x+2=0$

$$\begin{array}{r|rrrr} & -1 & 0 & 3 & 2 \\ -1 & & 1 & -1 & -2 \\ \hline & -1 & 1 & 2 & \underline{|0} \\ -1 & & 1 & -2 & \\ \hline & -1 & 2 & \underline{|0} & \\ 2 & & -2 & & \\ \hline & -1 & \underline{|0} & & \end{array}$$

$x = \begin{cases} -1 \text{ (doble)} \\ 2 \end{cases}$

b) • $\boxed{\text{Si } \left.\begin{matrix} x \neq -1 \\ x \neq 2 \end{matrix}\right\} \rightarrow \text{rango}(A) = 3}$

• Si $x=-1$ : $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \rightarrow$ obviamente : $\boxed{\text{si } x=-1 \rightarrow \text{rango}(A)=1}$

• Si $x=2$ : $A = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$ $\left.\begin{matrix} \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \neq 0 \\ \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = 0 \end{matrix}\right\} \Rightarrow \boxed{\text{si } x=2 \rightarrow \text{rango}(A)=2}$

SEPT10 fase general

a) $|A| = \begin{vmatrix} 1 & m & 0 \\ 0 & 1 & m \\ 1 & 1 & -2 \end{vmatrix} = -2+m^2-m = m^2-m-2$

b) $m^2-m-2=0$

$m = \dfrac{1 \pm \sqrt{1+8}}{2} = \dfrac{1 \pm 3}{2} \begin{matrix} \nearrow 2 \\ \searrow -1 \end{matrix}$

$\boxed{\text{Si } \left.\begin{matrix} m \neq 2 \\ m \neq -1 \end{matrix}\right\} \Rightarrow A \text{ tiene inversa}}$

c) $m=1$ : $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{vmatrix} = -2+1-1 = -2$

$A_{11} = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -3$ | $A_{21} = -\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = 2$ | $A_{31} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 1$ | $A_{22} = \begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = -2$ | $A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$ | $A_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$ | $A_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$$\boxed{A^{-1} = \frac{1}{-2} \begin{pmatrix} -3 & 2 & 1 \\ 1 & -2 & -1 \\ -1 & 0 & 1 \end{pmatrix}}$$

SEP10 fase específica

a) $|M| = \begin{vmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{vmatrix} = -2-2+1+4 = 1 \neq 0 \Rightarrow \exists M^{-1}$

$M_{11} = \begin{vmatrix} 1 & 1 \\ -2 & -2 \end{vmatrix} = 0$ | $M_{21} = -\begin{vmatrix} 2 & 1 \\ -2 & -2 \end{vmatrix} = 2$ | $M_{31} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$M_{12} = -\begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = 1$ | $M_{22} = \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} = 1$ | $M_{32} = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$M_{13} = \begin{vmatrix} 1 & 1 \\ -1 & -2 \end{vmatrix} = -1$ | $M_{23} = -\begin{vmatrix} 0 & 2 \\ -1 & -2 \end{vmatrix} = -2$ | $M_{33} = \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = -2$

$$\boxed{M^{-1} = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{pmatrix}}$$

b) $XM + M = 2M^2$ ; $XM = 2M^2 - M$ ; $X = (2M^2-M)M^{-1} = 2M - I$

$$X = 2 \cdot \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} -1 & 4 & 2 \\ 2 & 1 & 2 \\ -2 & -4 & -5 \end{pmatrix}}$$

JUN 11 fase general

a) $|A| = \begin{vmatrix} a+1 & 2 & a+1 \\ 0 & a-1 & 1-a \\ 1 & 1 & a \end{vmatrix} = (a+1)(a-1)a + 2(1-a) - (a+1)(a-1) - (a+1)(1-a) =$

$1-a = -(a-1)$

$= (a+1)(a-1)a - 2(a-1) - \cancel{(a+1)(a-1)} + \cancel{(a+1)(a-1)} =$

$= (a-1)\cdot\big((a+1)a-2\big) = (a-1)(a^2+a-2) = (a-1)(a-1)(a+2) = \boxed{(a-1)^2(a+2)}$

$|A|=0 \Rightarrow (a-1)^2(a+2)=0 \; : \; a = \begin{cases} 1 \\ 2 \end{cases}$

$\boxed{A \text{ tiene inversa para } \begin{cases} a\neq 1 \\ a\neq 2 \end{cases}}$

b) $\underline{\underline{a=0}}$ : $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2+1-1 = 2 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{21} = -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{31} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = +1 \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 \quad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$

$A^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 3 \\ 1 & -1 & -1 \\ +1 & 1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} -1/2 & 1/2 & 3/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}}$

JUN 11 fase específica

a) $|A| = \begin{vmatrix} 1 & a & 1 \\ 1 & 1 & 0 \\ 1 & a & 0 \end{vmatrix} = a-1$

$|A|=0 \Rightarrow a=1$

$\boxed{A \text{ tiene inversa para } a\neq 1}$

b) $A_{11} = \begin{vmatrix} 1 & 0 \\ a & 0 \end{vmatrix} = 0 \quad A_{21} = -\begin{vmatrix} a & 1 \\ a & 0 \end{vmatrix} = a \quad A_{31} = \begin{vmatrix} a & 1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{13} = \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = a-1 \quad A_{23} = -\begin{vmatrix} 1 & a \\ 1 & a \end{vmatrix} = 0 \quad A_{33} = \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix} = 1-a$

$A^{-1} = \frac{1}{a-1}\begin{pmatrix} 0 & a & -1 \\ 0 & -1 & 1 \\ a-1 & 0 & 1-a \end{pmatrix} = \boxed{\begin{pmatrix} 0 & a/(a-1) & -1/(a-1) \\ 0 & -1/(a-1) & 1/(a-1) \\ 1 & 0 & -1 \end{pmatrix}}$ (para $a\neq 1$)

Julio 11 fase específica

$AX=B \Rightarrow X = A^{-1}B$ si es que existe $A^{-1}$

$|A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = 2+2+1 = 5 \neq 0 \Rightarrow \exists A^{-1}$ ✓

$A_{11} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \quad A_{21} = -\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \quad A_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} = 2 \quad A_{22} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3 \quad A_{32} = -\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \quad A_{23} = -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1 \quad A_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$

$\boxed{A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{pmatrix}}$

$X = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ -1 & 1 & 2 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 4 & 6 & -2 \\ -2 & -3 & 1 \\ 1 & 4 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 4/5 & 6/5 & -2/5 \\ -2/5 & -3/5 & 1/5 \\ 1/5 & 4/5 & 2/5 \end{pmatrix}}$

JUN 12
fase
específica

a) $|A| = \begin{vmatrix} a-1 & 2 & a-1 \\ 0 & a+1 & -1-a \\ 1 & 1 & a \end{vmatrix} = (a-1)(a+1)a + 2(-1-a) - (a+1)(a-1) - (a-1)(-1-a) =$

$-1-a = -(a+1)$

$= (a-1)(a+1)a - 2(a+1) - \cancel{(a+1)(a-1)} + \cancel{(a-1)(a+1)} =$

$= (a+1)\big((a-1)a-2\big) = (a+1)(a^2-a-2) = (a+1)(a+1)(a-2) = (a+1)^2(a-2)$

$|A|=0 \Rightarrow (a+1)^2(a-2)=0 \quad ; \quad a = \begin{cases} -1 \\ 2 \end{cases}$

$\boxed{A \text{ tiene inversa para } \begin{cases} a \neq -1 \\ a \neq 2 \end{cases}}$

b) $\underline{a=0}$ : $A = \begin{pmatrix} -1 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = \begin{vmatrix} -1 & 2 & -1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = -2+1-1 = -2 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \qquad A_{21} = -\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1 \qquad A_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1$

$A_{12} = -\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = -1 \qquad A_{22} = \begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} = 1 \qquad A_{32} = -\begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix} = -1$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1 \qquad A_{23} = -\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} = 3 \qquad A_{33} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -1$

$A^{-1} = \frac{1}{-2}\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & 3 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & -3/2 & 1/2 \end{pmatrix}}$

Julio 12
fase
general

$A = \begin{pmatrix} x & b & c-4 \\ a & x & 3 \\ b & c & x \end{pmatrix}$

a) $A^t = -A \Rightarrow \begin{cases} x=0 \\ b=-a \\ c-4=-b \\ x=0 \\ 3=-c \\ x=0 \end{cases} \Rightarrow \begin{cases} \boxed{x=0} \\ a+b=0 \longrightarrow a=-b=\boxed{-7} \\ b+c=4 \longrightarrow b=4-c=\boxed{7} \\ \boxed{c=-3} \end{cases}$

b) $\left.\begin{matrix} a=1 \\ b=1 \\ c=1 \end{matrix}\right| \longrightarrow A = \begin{pmatrix} x & 1 & -3 \\ 1 & x & 3 \\ 1 & 1 & x \end{pmatrix}$

$|A| = \begin{vmatrix} x & 1 & -3 \\ 1 & x & 3 \\ 1 & 1 & x \end{vmatrix} = x^3 \cancel{-3} \cancel{+3} \cancel{+3x} - x \cancel{-3x} = x^3 - x = x(x^2-1) = x(x+1)(x-1)$

$|A|=0 \to x = \begin{cases} 0 \\ 1 \\ -1 \end{cases}$

- $\boxed{\text{Si } x \neq \begin{cases} 0 \\ 1 \\ -1 \end{cases} \Rightarrow \text{rango}(A)=3}$

- $x=0$ : $A = \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \end{pmatrix}$

$|A| = 0-3+3 = 0 \Rightarrow \text{rango}(A) < 3$

$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 \neq 0 \Rightarrow \text{rango}(A) = 2$

$\boxed{\text{Si } x=0 \Rightarrow \text{rango}(A)=2}$

• $x = 1$: $\quad A = \begin{pmatrix} 1 & 1 & -3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & -3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \cancel{1} - \cancel{3} + \cancel{3} + \cancel{3} - \cancel{1} - \cancel{3} = 0 \longrightarrow \text{rango}(A) < 3$$

$$\begin{vmatrix} 1 & -3 \\ 1 & 3 \end{vmatrix} = 3 + 3 = 6 \neq 0 \Rightarrow \text{rango}(A) = 2$$

$\boxed{\text{Si } x = 1 \Rightarrow \text{rango}(A) = 2}$

• Si $x = -1$: $\quad A = \begin{pmatrix} -1 & 1 & -3 \\ 1 & -1 & 3 \\ 1 & 1 & -1 \end{pmatrix}$

$$|A| = \begin{vmatrix} -1 & 1 & -3 \\ 1 & -1 & 3 \\ 1 & 1 & -1 \end{vmatrix} = -\cancel{1} - \cancel{3} + \cancel{3} - \cancel{3} + \cancel{1} + \cancel{3} = 0 \rightarrow \text{rango}(A) < 3$$

$$\begin{vmatrix} -1 & 3 \\ 1 & -1 \end{vmatrix} = 1 - 3 = -2 \neq 0 \Rightarrow \text{rango}(A) = 2$$

$\boxed{\text{Si } x = -1 \Rightarrow \text{rango}(A) = 2}$

c) $\left.\begin{matrix} a = 0 \\ b = 0 \\ c = 0 \end{matrix}\right| \rightarrow A = \begin{pmatrix} x & 0 & -4 \\ 0 & x & 3 \\ 0 & 0 & x \end{pmatrix}$

$$A + A^t = \begin{pmatrix} x & 0 & -4 \\ 0 & x & 3 \\ 0 & 0 & x \end{pmatrix} + \begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ -4 & 3 & x \end{pmatrix} = \begin{pmatrix} 2x & 0 & -4 \\ 0 & 2x & 3 \\ -4 & 3 & 2x \end{pmatrix}$$

$$|A + A^t| = \begin{vmatrix} 2x & 0 & -4 \\ 0 & 2x & 3 \\ -4 & 3 & 2x \end{vmatrix} = 8x^3 - 32x - 18x = 8x^3 - 50x = 2x(4x^2 - 25)$$

$$|A + A^t| = 0 \Rightarrow 2x(4x^2 - 25) = 0 \Rightarrow \boxed{x \begin{cases} 0 \\ \pm 5/2 \end{cases}}$$

Julio 12 fase específica

a) $|A| = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 2 \neq 0 \Rightarrow \exists A^{-1}$

$A_{11} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \qquad A_{21} = -\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \qquad A_{31} = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 3$

$A_{12} = -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} = 0 \qquad A_{22} = \begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = 2 \qquad A_{32} = -\begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4$

$A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \qquad A_{23} = -\begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix} = 0 \qquad A_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$

$$\boxed{A^{-1} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -4 \\ 0 & 0 & 2 \end{pmatrix}}$$

b) $XA = B \Rightarrow X = B \cdot A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \cdot \frac{1}{2} \begin{pmatrix} 1 & -1 & 3 \\ 0 & 2 & -4 \\ 0 & 0 & 2 \end{pmatrix} =$

$$= \frac{1}{2} \begin{pmatrix} 1 & -3 & 7 \\ 2 & -2 & 12 \end{pmatrix} = \boxed{\begin{pmatrix} 1/2 & -3/2 & 7/2 \\ 1 & -1 & 6 \end{pmatrix}}$$

Julio 12 fase específica

a) $|A| = \begin{vmatrix} -x & 1 & 0 \\ 0 & -x & 1 \\ c & b & a-x \end{vmatrix} = x^2(a-x) + c + bx = \boxed{-x^3 + ax^2 + bx + c}$

b) $c = 0 \rightarrow p(x) = -x^3 + ax^2 + bx$

$-x^3 + ax^2 + bx = 0 \;;\quad x(-x^2 + ax + b) = 0 \rightarrow \boxed{x = 0}$

$\rightarrow -x^2 + ax + b = 0$

$$x = \boxed{\frac{-a \pm \sqrt{a^2 + 4b}}{-2a}}$$

Junio 13
fase
General

$$A=\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}$$

$$M=A^2-A^t=\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1&a&1\\1&1&0\\1&0&0\end{pmatrix}-\begin{pmatrix}1&1&1\\a&1&0\\1&0&0\end{pmatrix}=$$

$$=\begin{pmatrix}a+2&2a&1\\2&a+1&1\\1&a&1\end{pmatrix}-\begin{pmatrix}1&1&1\\a&1&0\\1&0&0\end{pmatrix}=\begin{pmatrix}a+1&2a-1&0\\2-a&a&1\\0&a&1\end{pmatrix}$$

$$|M|=\begin{vmatrix}a+1&2a-1&0\\2-a&a&1\\0&a&1\end{vmatrix}=a(\cancel{a+1})-(2-a)(2a-1)-a(\cancel{a+1})=(a-2)(2a-1)$$

$$|M|=0\Rightarrow\boxed{a=\begin{cases}2\\1/2\end{cases}}$$

- Si $\left.\begin{array}{l}a\neq 2\\a\neq 1/2\end{array}\right|\Rightarrow r(M)=3$
- Si $a=2$ : $M=\begin{pmatrix}3&3&0\\0&2&1\\0&2&1\end{pmatrix}$ $\quad\begin{vmatrix}3&3\\0&2\end{vmatrix}=6\neq0\Rightarrow r(M)=2$
- Si $a=1/2$ : $M=\begin{pmatrix}3/2&0&0\\3/2&1/2&1\\0&1/2&1\end{pmatrix}$ $\quad\begin{vmatrix}3/2&0\\3/2&1/2\end{vmatrix}=\frac{3}{4}\neq0\Rightarrow r(M)=2$

JUN 13
fase
Específica

$$|A|=\begin{vmatrix}1&a&1\\1-a&1&2\\a&a^2&-1\end{vmatrix}=-1+a^2(1-a)+2a^2-a+a(1-a)-2a^2=$$

$$=-1+\cancel{a^2}-a^3+\cancel{2a^2}-\cancel{a}+\cancel{a}-\cancel{a^2}-\cancel{2a^2}=-1-a^3$$

$$|A|=0\Rightarrow -1-a^3=0\ ;\quad a^3=-1\ ;\quad a=\sqrt[3]{-1}=-1$$

a) • Si $a=-1$, no existe $A^{-1}$

• Si $a\neq-1$, sí existe $A^{-1}$

b) $\underline{\underline{a=0}}$ $\quad A=\begin{pmatrix}1&0&1\\1&1&2\\0&0&-1\end{pmatrix}$

$$|A|=\begin{vmatrix}1&0&1\\1&1&2\\0&0&-1\end{vmatrix}=-1$$

$A_{11}=\begin{vmatrix}1&2\\0&-1\end{vmatrix}=-1$ $\quad A_{12}=-\begin{vmatrix}1&2\\0&-1\end{vmatrix}=1$ $\quad A_{13}=\begin{vmatrix}1&1\\0&0\end{vmatrix}=0$

$A_{21}=-\begin{vmatrix}0&1\\0&-1\end{vmatrix}=0$ $\quad A_{22}=\begin{vmatrix}1&1\\0&-1\end{vmatrix}=-1$ $\quad A_{23}=-\begin{vmatrix}1&0\\0&0\end{vmatrix}=0$

$A_{31}=\begin{vmatrix}0&1\\1&2\end{vmatrix}=-1$ $\quad A_{32}=-\begin{vmatrix}1&1\\1&2\end{vmatrix}=-1$ $\quad A_{33}=\begin{vmatrix}1&0\\1&1\end{vmatrix}=1$

$$A^{-1}=\frac{1}{-1}\begin{pmatrix}-1&0&-1\\1&-1&-1\\0&0&1\end{pmatrix}=\boxed{\begin{pmatrix}1&0&1\\-1&1&1\\0&0&-1\end{pmatrix}}$$

JUL 13 fase General

$$|A| = \begin{vmatrix} -a & 2 & 0 \\ 1 & -1-a & 0 \\ 0 & 0 & 1-a \end{vmatrix} = -a(-1-a)(1-a) - 2(1-a) = (1-a)\cdot[a+a^2-2].$$

$$|A|=0 \Rightarrow \begin{cases} 1-a=0 \Rightarrow a=1 \\ a^2+a-2=0 \Rightarrow a=\begin{cases}1\\-2\end{cases}\end{cases}$$

JUL 13 fase Específ

$$A - x I_3 = \begin{pmatrix} 0 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - x \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -x & 2 & 0 \\ 1 & -1-x & 0 \\ 0 & 0 & 1-x \end{pmatrix}$$

$$p(x) = |A - x I_3| = \begin{vmatrix} -x & 2 & 0 \\ 1 & -1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = -x(-1-x)(1-x) - 2(1-x) =$$

$$= (1-x)[x+x^2-2] = -(x-1)(x^2+x-2)$$

$$p(x)=0 \Rightarrow \begin{cases} x-1=0 \Rightarrow x=1 \\ x^2+x-2=0 \Rightarrow x=\frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} = \begin{cases}1\\-2\end{cases}\end{cases}$$

$$p(x) = -(x-1)(x-1)(x+2) = \boxed{-(x-1)^2(x+2)}$$